# Physics 331 – Final Exam

This exam is due at noon on Wednesday, March 16. Please hand it in to Alex Saltman in Varian 363. If you have any questions about the exam, please contact me at mpeskin@slac.stanford.edu or 926-3250. If errata are reported, I will announce them on the course Web page.

Please do not collaborate on this exam. Please return the exam in a blue book (or multiple blue books) with the honor code acknowledgement signed. Alex Saltman has blue books, if you need one.

The exam is worth a total of 100 points. The distribution of points is indicated below.

I recommend that you try simply to answer the questions using methods from Physics 331. However, the exam is open-book. If you find a useful reference other than the class textbook and notes, feel free to use it, as long as you cite the reference in your solution.

Consider an $SU(3) \times SU(2) \times U(1)$ gauge theory. Fermions and scalars in this theory are in gauge representations $(R, I, Y)$, where $R$ is the $SU(3)$ representation, $I$ is the $SU(2)$ representation, and $Y$ is the $U(1)$ charge. Consider a Higgs boson boson $\Sigma$ in the represention $(3, 2, -\frac{1}{2})$. This can be written as a $3 \times 2$ matrix of complex fields. The covariant derivative on this multiplet is

$$D_\mu \Sigma = \partial_\mu \Sigma - i g_3 \mathcal{A}^A_\mu t^A \Sigma + i g_2 \mathcal{B}^a_\mu \Sigma \tau^a - i g_1 \frac{1}{2} \mathcal{C}_\mu \Sigma \tag{1}$$

where $t^A$ are the usual $SU(3)$ generators and $\tau^a$ are the usual $SU(2)$ generators. Note that the theory has three independent coupling constants $g_1$, $g_2$, $g_3$.

a. (10 points) Show that

$$\langle \Sigma \rangle = V \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \tag{2}$$

spontaneously breaks $SU(3) \times SU(2) \times U(1)$ to $SU(2) \times U(1)$. Write the unbroken gauge group generators explicitly and show that they annihilate $\langle \Sigma \rangle$.

b. (20 points) Work out the mass matrix for the vectors bosons due to the Higgs mechanism. Write explicitly the linear combinations of the gauge bosons in (1) that are mass eigenstates. Find the quantum numbers of each under the unbroken $SU(2) \times U(1)$. Compute the masses of the massive bosons.

c. (10 points) Introduce quarks and leptons into the model. In the Standard Model, quarks and leptons belong to representations $(I, Y)$ of $SU(2) \times U(1)$. For example, the left-handed quark doublet $Q_L$ is a $(2, \frac{1}{6})$. In this model, we can put quarks and leptons into the corresponding representations $(1, I, Y)$, e.g., $(1, 2, \frac{1}{6})$ for $Q_L$. Write the covariant derivative acting on quark and lepton multiplets in terms of the mass eigenstate boson fields. Note that the couplings of the massless $SU(2) \times U(1)$ gauge bosons are functions of $g_1$, $g_2$, $g_3$. Show that, if $g_1, g_2 \gg g_3$, the model predicts $\sin^2\theta_w = \frac{1}{4}$ at the scale $Q = M_\Sigma$ of the spontaneous breaking.

d. (15 points) Using the equations for the running of the $SU(2)$ and $U(1)$ couplings, compute the $SU(2)$ and $U(1)$ gauge couplings at values of $Q$ lower than $M$. If

$$\frac{g^2}{4\pi} = \frac{1}{29.6} \qquad \frac{g'^2}{4\pi} = \frac{1}{98.5} \tag{3}$$

at $Q = m_Z$—and still assuming $g_1, g_2 \gg g_3$—find $g_3$ and $M_\Sigma$. For simplicity, treat the top quark as approximately massless in this analysis.

e. (15 points) How many of the heavy gauge bosons are singly produced in the Drell-Yan process $q\overline{q} \to V$? For each of these, write the parton model expression for the total cross section

$$\sigma(pp \to V + X) \tag{4}$$

f. (10 points) Write the parton model formula for the neutrino deep inelastic scatting cross section

$$\frac{d^2\sigma}{dxdy}(\nu p \to \mu^- + X) \tag{5}$$

in this model. How accurately do we have to measure this cross section to be sensitive to the effects of the exchanges of the heavy vector bosons?

g. (20 points) The heavy charged boson $\mathcal{W}^+$ can decay either to a pair of fermions or to a pair of gauge bosons. List the possible decay products in each case, and compute the partial widths in terms of $g_1$, $g_2$, $g_3$. Which set of decays is more important?