Physics 330 – Quiz

(issued Friday, October 31; due Wednesday, November 5)

I will use the quiz and the final in this course to assign grades for this course, keeping in mind that anyone who hands in all of the problem sets is assured a reasonable grade. Because the quiz will be graded, please abide by these rules:

- The quiz is open-book. You may use any reference resources that you find. You may use your mathematical software such as MatLab or Mathematica. However, please do not collaborate with other students or ask help from other humans—except that, if you have any questions about the quiz, please feel free to email me: mpeskin@slac.stanford.edu. If you find an AI that can solve this quiz, please let me know about it.
- The quiz is posted at the course web site:

  `https://s3df.slac.stanford.edu/people/mpeskin/Physics330/`

  This is the cover page. Please hand in your solution (upload to Gradescope) within 24 hours of the time that you turn the page and begin to solve the quiz.
- Please write on your solution: "I acknowledge the Stanford Honor Code." and sign it.
- The quizzes should be turned in to Gradescope by the due date above. If this is a problem for you, please email me to ask for another arrangement.

The quiz has 1 long problem and will be worth 30 points. Partial credit will be given. (The final will be worth 70 points.)

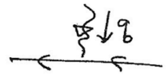
1. In Problem Set 2, we worked out that the Dirac equation leads to a magnetic moment for the Dirac particle with $g = 2$. However, we could add a term to the Dirac Hamiltonian that modifies the values of g. From the results of that problem, the Hamiltonian

$$H = H_{QED} + \int d^3x \; e\frac{a}{4m}\; \overline{\psi}\, \sigma^{\mu\nu} F_{\mu\nu}\, \psi, \tag{1}$$

where $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$, would give a Landé g factor

$$g = 2(1 + a)\ . \tag{2}$$

The Feynman rule for this new vertex is



$$= \; e\,\frac{a}{2m}\,\sigma^{\mu\nu} q_\nu \tag{3}$$

Today, the measured magnetic moments of the leptons are all in good agreement with the predictions of QED. The magnetic moments of the electron and muon are measured by very special methods that give precisions better than 1 part per billion and 1 part per million, respectively. However, the magnetic moment of the tau lepton is only known to about 1% and is extracted from the measurement of basic QED reactions. So it is interesting to work out the effect of this new term on the angular distribution of $e^+e^- \to \tau^+\tau^-$. In the following, we will consider only the cross section for unpolarized beams, summed over final spin orientations. You can set the mass of the electron equal to 0.

(a) In class, we found that, for leading-order QED, in the CM frame,

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s}\sqrt{1 - \frac{4m_\tau^2}{s}} \cdot (1 + \cos^2\theta + \frac{4m_\tau^2}{s}\sin^2\theta)\ , \tag{4}$$

where $s = E_{CM}^2$. Compute the correction to this formula linear in a.

(b) Compute the term in this formula proportional to a^2. Since a is known to be small, these terms are important only at high energy (high values of s), so you can drop terms in the numerator proportation to m.

(c) Why is the a^2 term so much more important than the a term at high energy? Is this behavior physical?