Physics 330 – Quiz # 1

(issued Thursday, October 31; due Wednesday, November 6)

I will use the quiz and the final in this course to assign grades for this course, keeping in mind that anyone who hands in all of the problem sets is assured a reasonable grade. Because the quiz will be graded, please abide by these rules:

- The quiz is open-book. You may use any reference resources that you find. You may use your mathematical software such as MatLab or Mathematica. However, please do not collaborate with other students or ask help from other humans—except that, if you have any questions about the quiz, please feel free to email me: mpeskin@slac.stanford.edu. If you find an AI that can solve this quiz, please let me know about it.
- The quiz is posted at the course web site:

  `https://www.slac.stanford.edu/ mpeskin/Physics330/`

  This is the cover page. Please hand in your solution (upload to Gradescope) within 24 hours of the time that you turn the page and begin to solve the quiz.
- Please write on your solution: "I acknowledge the Stanford Honor Code." and sign it.
- The quizzes should be turned in to Gradescope by the due date above. If this is a problem for you, please email me to ask for another arrangement.

The quiz has 1 long problem and will be worth 30 points. Partial credit will be given. (The final will be worth 70 points.)

1. How do you measure the polarization of a polarized electron beam? One way is to use electron-electron scattering (Møller scattering). Shoot the electron beam into a fully magnetized iron foil, and measure the asymmetry in the scattering rate when the direction of magnetization, or the polarization of the beam, is flipped.

   (a) Draw the Feynman diagrams for electron-electron scattering in QED, at leading order. Write the expression for the scattering amplitude in terms of $u(p)$ spinors. Remember that electrons are identical particles.

   (b) Consider scattering through an angle θ. Work the in the center of mass frame, and use the approximation that the electrons are very relativistic, so that you can ignore the electron mass. Then you can consider the initial and final electrons to be left (L)- or right (R)-handed, and use helicity conservation. Work out the needed current matrix elements $\overline{u}(p')\gamma^\mu u(p)$ as 4-vectors depending on the CM electron energy E and the scattering angle $\cos\theta$.

   (c) By dotting these 4-vectors together, compute the scattering amplitude for each of the possible initial polarization combinations:

   $$e^-(R)e^-(R)\ , \quad e^-(L)e^-(R)\ , \quad e^-(R)e^-(L)\ , \quad e^-(L)e^-(L)\ . \tag{1}$$

   You can check your work by using the fact that QED is invariant under parity. Don't forget the fermion minus sign!

   (d) The polarization asymmetry is defined as

   $$\mathcal{A}_{LR} = \frac{d\sigma/d\cos\theta(e^-(R)e^-(R)) - d\sigma/d\cos\theta(e^-(L)e^-(R))}{d\sigma/d\cos\theta(e^-(R)e^-(R)) + d\sigma/d\cos\theta(e^-(L)e^-(R))} \tag{2}$$

   (By parity, the asymmetry should be equal for all combinations, $RR - LR$, $RR - RL$, $LL - RL$, $LL - LR$.) The measured event rate asymmetry is then

   $$\mathcal{A}_{meas.} = P_{beam}\, P_{foil}\, \mathcal{A}_{LR}\ , \tag{3}$$

   where P_{beam} is the polarization of the beam and P_{foil} is the polarization of the electrons in the foil. Compute $\mathcal{A}_{LR}$ as a function of $\cos\theta$.

   (e) Show that the asymmetry $\mathcal{A}_{LR}$ is maximized at $\cos\theta = 0$, i.e., scattering at 90°.

   (f) Give a physical argument why the polarization asymmetry $\mathcal{A}_{LR}$ should vanish for forward scattering ($\cos\theta = 1$).

   (g) In the actual experiment, a 45 GeV electron beam is shot at a foil in rest in the lab. What lab angle corresponds to $\cos\theta = 0$? (It is a pretty small angle.) A better way to do the experiment is to momentum-analyze the scattered electron and select the lab momentum that corresponds to scattering by 90° in the CM frame. What is the value of this momentum?