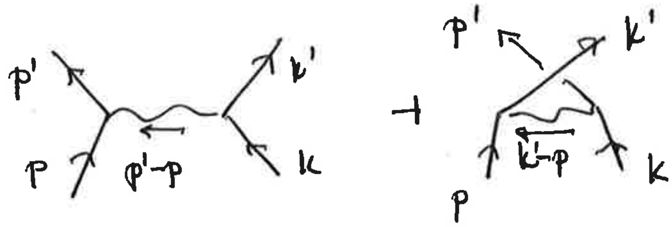
Physics 330 — Quiz

Solutions

a)

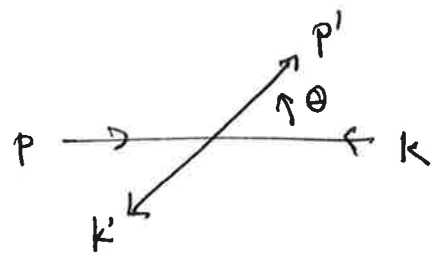


$$i\mathcal{M} = (-ie)^2 \Big\{ \bar{u}(p')\gamma^\mu u(p) \frac{-ig_{\mu\nu}}{(p'-p)^2} \bar{u}(k')\gamma^\nu u(k)$$

$$- \bar{u}(k')\gamma^\mu u(p) \frac{-ig_{\mu\nu}}{(k'-p)^2} \bar{u}(p')\gamma^\nu u(k) \Big\}$$

fermion interchange

b.) Let's first write out the spinors. The kinematics is



in the limit $E \gg m_e$. in the CM frame.

$p = (E, 0, 0, E)$ $\qquad p' = (E, Es, 0, Ec)$

$k = (E, 0, 0, -E)$ $\qquad k' = (E, -Es, 0, -Ec)$

$s = \sin\theta \quad c = \cos\theta$

initial state spinors

helicity $-\frac{1}{2}$: $\quad u_L(p) = \sqrt{2E}\begin{pmatrix} 0 \\ 1 \\ \hline 0 \\ 0 \end{pmatrix} \qquad u_L(k) = \sqrt{2E}\begin{pmatrix} 1 \\ 0 \\ \hline 0 \\ 0 \end{pmatrix}$

helicity $+\frac{1}{2}$: $\quad u_R(p) = \sqrt{2E}\begin{pmatrix} 0 \\ 0 \\ \hline 1 \\ 0 \end{pmatrix} \qquad u_R(k) = \sqrt{2E}\begin{pmatrix} 0 \\ 0 \\ \hline 0 \\ -1 \end{pmatrix}$

final state spinors (rotate the above through θ:

helicity $-\frac{1}{2}$: $\quad u_L(p') = \sqrt{2E}\begin{pmatrix} -s_2 \\ c_2 \\ \hline 0 \\ 0 \end{pmatrix} \qquad u_L(k') = \sqrt{2E}\begin{pmatrix} c_2 \\ s_2 \\ \hline 0 \\ 0 \end{pmatrix}$

helicity $+\frac{1}{2}$: $\quad u_R(p') = \sqrt{2E}\begin{pmatrix} 0 \\ 0 \\ \hline c_2 \\ s_2 \end{pmatrix} \qquad u_R(k') = \sqrt{2E}\begin{pmatrix} 0 \\ 0 \\ \hline -s_2 \\ c_2 \end{pmatrix}$

$$s_2 = \sin\theta/2 \qquad c_2 = \cos\theta/2$$

For left-hand spinors $\quad u = \sqrt{2E}\begin{pmatrix} \xi \\ 0 \end{pmatrix}$

$$\bar{u}_L \gamma^\mu u_L' = 2E\,(\xi^\dagger, 0)\,\gamma^0\gamma^\mu \begin{pmatrix} \xi' \\ 0 \end{pmatrix}$$

$$= 2E\ \xi^\dagger \bar{\sigma}^\mu \xi'$$

For right-handed spinors $\quad u = \sqrt{2E}\begin{pmatrix} 0 \\ \xi \end{pmatrix}$

$$\bar{u}_R \gamma^\mu u_R' = 2E\,(0\ \xi^\dagger)\,\gamma^0\gamma^\mu \begin{pmatrix} 0 \\ \xi' \end{pmatrix}$$

$$= 2E\ \xi^\dagger \sigma^\mu \xi'$$

and

$$\bar{u}_L \gamma^\mu u_R = \bar{u}_R \gamma^\mu u_L = 0$$

then $\sigma^\mu = (1, \vec{\sigma}) \quad \bar{\sigma}^\mu = (1, -\vec{\sigma})$

$$\bar{u}_L(p')\gamma^\mu u_L(p) = 2E\,(-s_2\, c_2)\,(1, -\vec{\sigma})\begin{pmatrix}0\\1\end{pmatrix}$$
$$= 2E\,(c_2, s_2, -is_2, c_2)^\mu$$

$$\bar{u}_L(k')\gamma^\mu u_L(k) = 2E\,(c_2, s_2)\,(1, -\vec{\sigma})\begin{pmatrix}1\\0\end{pmatrix}$$
$$= 2E\,(c_2, -s_2, -is_2, -c_2)^\mu$$

$$\bar{u}_L(p')\gamma^\mu u_L(k) = 2E\,(-s_2\, c_2)\,(1, -\vec{\sigma})\begin{pmatrix}1\\0\end{pmatrix}$$
$$= 2E\,(s_2, -c_2, -ic_2, s_2)^\mu$$

$$\bar{u}_L(k')\gamma^\mu u(p) = 2E\,(c_2, s_2)\,(1, -\vec{\sigma})\begin{pmatrix}0\\1\end{pmatrix}$$
$$= 2E\,(-s_2, -c_2, +ic_2, +s_2)^\mu$$

<u>and</u>

$$\bar{u}_R(p')\gamma^\mu u_R(p) = 2E\,(c_2\, s_2)(1, \vec{\sigma})\begin{pmatrix}1\\0\end{pmatrix}$$
$$= 2E\,(c_2, s_2, is_2, c_2)$$

$$\bar{u}_R(k')\gamma^\mu u_R(k) = 2E\,(-s_2\, c_2)(1, \vec{\sigma})\begin{pmatrix}0\\1\end{pmatrix}$$
$$= 2E\,(c_2, -s_2, +is_2, -c_2)$$

$$\bar{u}_R(p')\gamma^\mu u_R(k) = 2E\,(c_2\, s_2)\,(1, \vec{\sigma})\begin{pmatrix}0\\1\end{pmatrix}$$
$$= 2E\,(s_2, c_2, -ic_2\ -s_2)$$

$$\bar{u}_R(k')\gamma^\mu u(p) = 2E\,(-s_2, c_2)\,(1, \vec{\sigma})\begin{pmatrix}1\\0\end{pmatrix}$$
$$= 2E\,(-s_2, c_2, ic_2, +s_2)$$

c.) Now compute the amplitude on p. 1 First of all, we need (ignoring m_e)

$$(p'-p)^2 = (p')^2 - 2pp' + p^2 = 0 - 2E^2(1-\cos\theta) + 0$$

$$= -2E^2(1-\cos\theta)$$

$$(k'-p)^2 = 0 - 2k'p + 0 = -2E^2(1+\cos\theta)$$

The various products of currents are:

$$\bar{u}_R(p')\gamma^\mu u_R(p)\ \bar{u}_R(k')\gamma_\mu u_R(k) = (2E)^2(c_2^2+s_2^2+s_2^2+c_2^2)$$

$$= 2(2E)^2$$

$$\bar{u}_R(p')\gamma^\mu u_R(p)\ \bar{u}_L(k')\gamma_\mu u_L(k) = (2E)^2(c_2^2+s_2^2-s_2^2+c_2^2)$$

$$= 2(2E)^2 c_2^2 = (2E)^2(1+c)$$

$$\bar{u}_L(p')\gamma^\mu u_L(p)\ \bar{u}_R(k')\gamma_\mu u_R(k) = (2E)^2(c_2^2+s_2^2-s_2^2+c_2^2)$$

$$= 2(2E)^2 c_2^2 = (2E)^2(1+c)$$

$$\bar{u}_L(p')\gamma^\mu u_L(p)\ \bar{u}_L(k')\gamma_\mu u_L(k) = (2E)^2(c_2^2+s_2^2+s_2^2+c_2^2)$$

$$= 2(2E)^2$$

$$\bar{u}_R(k')\gamma^\mu u_R(p)\ \bar{u}_R(p')\gamma_\mu u_R(k) = (2E)^2(-s_2^2-c_2^2-c_2^2-s_2^2)$$

$$= 2(2E)^2$$

$$\bar{u}_R(k')\gamma^\mu u_R(p)\ \bar{u}_L(p')\gamma_\mu u_L(k) = (2E)^2(-s_2^2+c_2^2-c_2^2-s_2^2)$$

$$= -2(2E)^2 s_2^2 = -(2E)^2(1-c)$$

$$\bar{u}_L(k')\gamma^\mu u_L(p)\ \bar{u}_R(p')\gamma_\mu u_R(k) = (2E)^2(-s_2^2 + c_2^2 - c_2^2 - s_2^2)$$
$$= -2(2E)^2 s_2^2 = -(2E)^2(1-c)$$

$$\bar{u}_L(k')\gamma^\mu u_L(p)\ \bar{u}_L(p')\gamma_\mu u_L(k) = (2E)^2(-s_2^2 - c_2^2 - c_2^2 - s_2^2)$$
$$= -2(2E)^2$$

and, any current of the form $\bar{u}_L\gamma^\mu u_R$ or $\bar{u}_R\gamma^\mu u_L = 0$

From $\bar{e}_R\bar{e}_R$ we can only get e_R^- into θ e_R^- into $(\pi-\theta)$

$$i\mathcal{M}_{RR} = ie^2\left\{\frac{2(2E)^2}{-2E^2(1-c)} - \frac{(-2)(2E)^2}{-2E^2(1+c)}\right\}$$

$$= -4ie^2\left(\frac{1}{1-c} + \frac{1}{1+c}\right) = -8ie^2\left(\frac{1}{1-c^2}\right)$$

From $\bar{e}_L e_R^-$ we can get e_L^- into θ e_R^- into $\pi-\theta$

$$i\mathcal{M}_{LR\to LR} = ie^2\left\{\frac{(2E)^2(1+c)}{-2E^2(1-c)} - 0\right\}$$

$$= -2ie^2\left(\frac{1+c}{1-c}\right)$$

and e_L^- into $\pi-\theta$ e_R^- into θ

$$i\mathcal{M}_{LR\to RL} = ie^2\left\{0 - \frac{-(2E)^2(1-c)}{-2E^2(1+c)}\right\}$$

$$= -2ie^2\left(\frac{1-c}{1+c}\right)$$

Similarly, for $e_R^- e_L^-$ we can get e_R^- into θ, e_L^- into $(\pi-\theta)$

$$i\mathcal{M}_{RL\to RL} = -2ie^2\left(\frac{1+c}{1-c}\right)$$

and e_R^- into $(\pi-\theta)$ e_L^- into θ

$$i\mathcal{M}_{RL\to LR} = -2ie^2\left(\frac{1-c}{1+c}\right)$$

for $e_L^- e_L^-$ we again get both diagrams interfering

$$i\mathcal{M}_{LL\to LL} = -8ie^2\left(\frac{1}{1-c^2}\right)$$

then

$$\frac{d\sigma}{d\cos\theta} = \frac{1}{2(2E)^2}\frac{1}{16\pi}|\mathcal{M}|^2$$

$$\frac{d\sigma}{d\cos\theta}(e_R^- e_R^-) = \frac{8\pi\alpha^2}{E^2}\frac{1}{(1-c^2)^2}$$

$$\frac{d\sigma}{d\cos\theta}(e_R^- e_L^-) = \frac{\pi\alpha^2}{2E^2}\left[\left(\frac{1+c}{1-c}\right)^2 + \left(\frac{1-c}{1+c}\right)^2\right]$$

$$= \frac{\pi\alpha^2}{2E^2}\frac{(1+c)^4+(1-c)^4}{(1-c^2)^2} = \frac{\pi\alpha^2}{E^2}\frac{1+6c^2+c^4}{(1-c^2)^2}$$

$$\frac{d\sigma}{d\cos\theta}(e_L^- e_R^-) = \frac{\pi\alpha^2}{E^2}\frac{1+6c^2+c^4}{(1-c^2)^2}$$

$$\frac{d\sigma}{d\cos\theta}(e_L^- e_L^-) = \frac{8\pi\alpha^2}{E^2}\frac{1}{(1-c^2)^2}$$

d.) The asymmetry is then

$$\sqrt{A_{LR}} = \frac{\frac{8}{(1-c^2)^2} - \frac{(1+6c^2+c^4)}{(1-c^2)^2}}{\frac{8}{(1-c^2)^2} + \frac{(1+6c^2+c^4)}{(1-c^2)^2}}$$

$$= \frac{7-6c^2-c^4}{9+6c^2+c^4} = \frac{(7+c^2)(1-c^2)}{(3+c^2)^2}$$

again

$$A_{LR} = \frac{(7+c^2)(1-c^2)}{(3+c^2)^2}$$

e.)

$$\frac{\partial}{\partial c} A_{LR} = \frac{2c}{(3+c^2)^2}\left[(1-c^2) - (7+c^2) - 2\frac{(7+c^2)(1-c^2)}{(3+c^2)}\right]$$

$$= \frac{2c}{(3+c^2)^3}\left[3-2c^2-c^4-21-10c^2-c^4 - 2(7-6c^2 - c^4)\right]$$

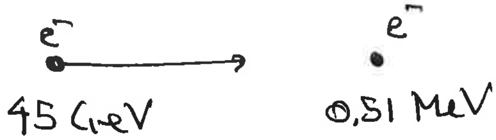
$$= \frac{2c}{(3+c^2)^3}[32]$$

this vanishes only at $c=0$

$\sqrt{A_{RR}}$ is positive and vanishes at $c=\pm 1$

So A_{LR} is maximized at $c=0$ $\quad 90^\circ$ scattering f.) Near $\cos\theta = 1$, the scattering is dominated by Coulomb scattering at low momentum transfer. This is spin-independent.

g.)



the center of mass energy is

$$E_{cm}^2 = (p+k)^2 = p^2 + 2p\cdot k + k^2$$

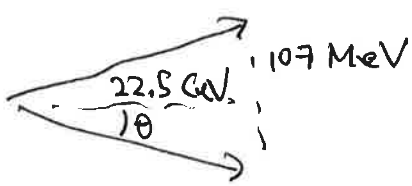
$$= 2p\cdot k + 2m_e^2$$

$$\approx 2p\cdot k = 2\,(45\times10^3)(0.51\text{ MeV})$$

$$= (214\text{ MeV})^2 \quad \text{or} \quad E = 107\text{ MeV}$$

so in the CM frame

~~boosting~~ back to the lab frame, the final electrons share the total momentum equally.



The lab momentum is 22.5 GeV.

so $\sin\theta = \dfrac{0.107}{22.5} \Rightarrow \theta = 4.7$ mrad or 0.27°