Physics 330 – Problem Set #6

Solutions

1.) a.) $\langle p'|S|p\rangle = \langle p'|T\{e^{-i\int dt\, H_I(t)}\}|p\rangle$

$$= \langle p'|1+iT|p\rangle \cong \langle p'|1|p\rangle + \langle p'| -i\int d^4x\, e\bar{\psi}_I\gamma^\mu\psi_I A_\mu(x)|p\rangle + \dots$$

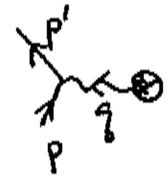
now set $\quad A_\mu(x) = \int \frac{d^4q}{(2\pi)^4} e^{-iq\cdot x}\tilde{A}_\mu(q)$

$$\psi_I|p\rangle = u(p)\, e^{-ip\cdot x}$$

$$\langle p'|\bar{\psi}_I = \bar{u}(p')\, e^{ip'\cdot x}$$

$\int d^4x$ gives $(2\pi)^4\delta(p'-p-q)$ so

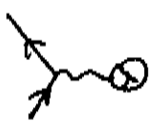
$$\langle p'|iT|p\rangle \cong -ie\,\bar{u}(p')\gamma^\mu u(p)\,\tilde{A}_\mu(q)\Big|_{q=p'-p} =$$



b.) For $A_\mu(x)$ independent of time x^0

$$A_\mu(x) = \int\frac{dq^0}{(2\pi)}\int\frac{d^3q}{(2\pi)^3} e^{-iq\cdot x} A_\mu(\vec{q})\,(2\pi)\delta(q^0)$$

this gives

$\langle p'|iT|p\rangle =$  $= -ie\,\bar{u}(p')\gamma^\mu u(p)\,\tilde{A}_\mu(\vec{q})\cdot 2\pi\,\delta(p'^0 - p^0)$

using this formula, we can derive the cross section from initial wave packets.

$$\int d\sigma = \int \frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f} \int d^2b \int \frac{d^3p_1}{(2\pi)^3}\frac{\phi(p_1)}{\sqrt{2E_1}} \int \frac{d^3p_2}{(2\pi)^3}\frac{\phi^*(p_2)}{\sqrt{2E_2}}\, e^{i\vec{b}\cdot(\vec{p}_1-\vec{p}_2)_\perp}$$

$$\cdot \quad \langle p' | iT | p_1\rangle \langle p_2 | -iT^\dagger | p'\rangle$$

the integral d^2b gives $(2\pi)^2\delta(\vec{p_1}-\vec{p_2})$ in the $\perp$ direction

The energy δ-function in the matrix elements set $p_1^3 = p_2^3$

$$\int \frac{dp_1^3}{(2\pi)}\, 2\pi\, \delta(E_f - E_1) = \frac{1}{\left|\frac{p_1}{E_1}\right|}$$

then

$$\int d\sigma = \int \frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f} \int \frac{d^3p_1}{(2\pi)^3}\frac{|\phi(p_1)|^2}{2E_1}\, |M|^2\, 2\pi\,\delta(E_f - E_1)\frac{1}{\left|\frac{p_1}{E_1}\right|}$$

$$\sigma = \frac{1}{2p}\int \frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}\,|M|^2\,(2\pi)\,\delta(E_f - E)$$

$$\frac{1}{2p} = \frac{1}{v\cdot 2E}$$

now $$\int \frac{dp_f\, p_f^2}{(2\pi)}\, 2\pi\,\delta(E_f - E) = \frac{p_f^2}{\left|\frac{p_f}{E_f}\right|}$$ so

$$\int d\sigma = \frac{1}{4p^2}\int \frac{d\Omega_f}{(2\pi)^2}\, p^2\, |M|^2 = \int d\Omega\, \frac{1}{16\pi^2}\, |M|^2$$

again $$\frac{d\sigma}{d\Omega} = \frac{1}{16\pi^2}|\mathcal{M}|^2$$

c.) For the Coulomb potential

$$A^0(\vec{q}) = \frac{Ze}{|\vec{q}|^2}$$

$$i\mathcal{M} = -ie\,\bar{u}(p')\gamma^0 u(p)\,A^0(\vec{q})$$

in the non-relativistic limit, $\bar{u}\gamma^0 u \approx 2m\,\delta_{ss'}$. Then

$$\frac{d\sigma}{d\Omega} = \frac{1}{16\pi^2}\,4m^2\,\frac{Z^2e^4}{(|\vec{q}|^2)^2}$$

$$|\vec{q}|^2 = |\vec{p}' - \vec{p}|^2 = 2p^2(1-\cos\theta) = 4p^2\sin^2\theta/2$$

$$= \frac{e^4}{16\pi^2}Z^2\,\frac{4m^2}{16p^4\sin^4\theta/2} \qquad p = mv$$

so $$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 Z^2}{4m^2v^4}\,\frac{1}{\sin^4\theta/2}$$

2.) Now redo this in the relativistic case

$$i\mathcal{M} = -ie\,\bar{u}(p')\gamma^0 u(p)\,A^0(\vec{q})$$

$$\frac{1}{2}\sum_{\text{spin}}|\mathcal{M}|^2 = \frac{1}{2}e^2\,\mathrm{tr}[(\not{p}'+m)\gamma^0(\not{p}+m)\gamma^0]\;|A^0(\vec{q})|^2$$

$$= \frac{1}{2}e^2\cdot 4\cdot[p'^0p^0\cdot 2 - p'\cdot p + m^2]\;|A^0(\vec{q})|^2$$

$$= 2e^2[\;2E^2 - (E^2 - p^2\cos\theta) + m^2]\;|A^0(\vec{q})|^2$$

$$= 2e^2[\;2E^2 - p^2(1-\cos\theta)\;]\;|A^0(\vec{q})|^2$$

$$= 4e^2E^2\left(1 - \left(\tfrac{p}{E}\right)^2\sin^2\theta/2\right)|A^0(\vec{q})|^2$$

for a Coulomb potential, still $A^0(\vec{q}) = \dfrac{Ze}{4p^2\sin^2\theta/2}$ $\qquad \beta = \frac{p}{E} = v$

so

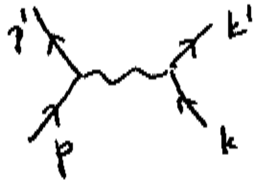
$$\frac{d\sigma}{d\Omega} = \frac{1}{16\pi^2}\,4e^4Z^2E^2(1-\beta^2\sin^2\theta/2)\,\frac{1}{16E^4\beta^4\sin^4\theta/2}$$

$$= \frac{\alpha^2Z^2}{4E^2\beta^4}\,\frac{1}{\sin^4\theta/2}\,(1-\beta^2\sin^2\theta/2)$$

or

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2Z^2}{4p^2\beta^2}\,\frac{1}{\sin^4\theta/2}\,(1-\beta^2\sin^2\theta/2)$$

Another way: use $e^-\mu^- \to e^-\mu^-$



$$= (-ie)^2\ \bar{u}(p')\gamma^\mu u(p)\ \frac{-i}{q^2}\ \bar{u}(k')\gamma_\mu u(k)$$

$$\frac{1}{4}\sum_{\text{spin}}|\mathcal{M}|^2 = \frac{e^4}{4q^4}\ \mathrm{tr}\left[(\not{p}'+m)\gamma^\mu(\not{p}+m)\gamma^\lambda\right]\ \mathrm{tr}\left[(\not{k}'+M)\gamma_\mu(\not{k}+M)\gamma_\lambda\right]$$

now

$$\mathrm{tr}\,(\not{k}'+M)\gamma^\mu(\not{k}+M)\gamma^\lambda$$

$$= 4\left[k'^\mu k^\lambda + k'^\lambda k^\mu - k\cdot k'\, g^{\mu\lambda} + M^2 g^{\mu\lambda}\right]$$

for $M\to\infty$ $\quad k \cong k' \cong (M,\vec{0}) \qquad k\cdot k' \approx M^2$

$$= 8M^2\,\delta^{\mu 0}\delta^{\lambda 0}$$

$$\frac{1}{4}\sum_{\text{spin}}|\mathcal{M}|^2 \cong \frac{e^4}{4q^4}\ \mathrm{tr}\left[(\not{p}'+m)\gamma^0(\not{p}+m)\gamma^0\right]\cdot 8M^2$$

$$\cong \frac{2e^4M^2}{q^4}\cdot 4E^2(1-\beta^2\sin^2\theta/2)$$

$$q^2 = -|\vec{q}|^2 = -4E^2\beta^2\sin^2\theta/2 \qquad q^4 = 16E^4\beta^4\sin^4\theta/2$$

so
$$\frac{1}{4}\sum_{\text{spin}}|\mathcal{M}|^2 \cong \frac{e^4 M^2}{E^2\beta^4\ \sin^4\theta/2}\,(1-\beta^2\sin^2\theta/2)$$

In the CM frame $\quad M \to \infty$

$$d\sigma = \frac{1}{2E \cdot 2M \cdot v} \; \frac{1}{8\pi} \; \frac{2p}{E_{cm}} \cdot \frac{d\Omega}{4\pi} \left(\frac{1}{4} \sum_{\text{spin}} |\mathcal{M}|^2 \right)$$

$$E_{cm} \approx M$$

$$= \frac{1}{16 \cdot \cancel{4} \pi^2} \; \frac{1}{M^2} \; d\Omega \left(\frac{1}{4} \sum |\mathcal{M}|^2 \right)$$

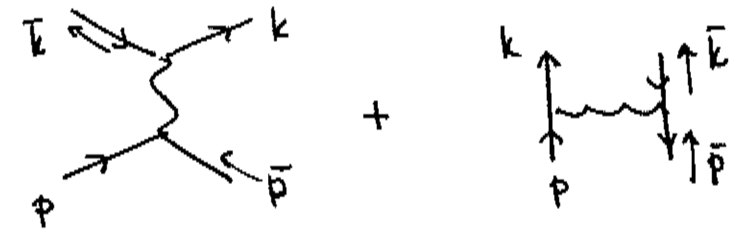
then

$$\frac{d\sigma}{d\Omega} = \frac{1}{16\pi^2 \cdot 4M^2} \cdot \frac{e^4 M^2}{E^2 \beta^4 \sin^4 \theta/2} \left(1 - \beta^2 \sin^2 \theta/2 \right)$$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 Z^2}{4 E^2 \beta^4 \sin^4 \theta/2} \left(1 - \beta^2 \sin^2 \theta/2 \right)$$

with $Z = 1$ in this case.

3.) The diagrams are:



 comes from $\langle \bar{k} k | \bar{\Psi}\psi \; \bar{\Psi}\psi \; | p \bar{p} \rangle \to +1$

 come from $\langle \bar{k} k | \bar{\Psi}\psi \; \bar{\Psi}\psi \; | p \bar{p} \rangle \to -1$ from fermion interchange

Ignoring e^- e^+ masses

so

$$i\mathcal{M} = (-ie)^2 \Big\{ \bar{u}(k)\gamma^\mu v(\bar{k}) \left(\frac{-i}{s}\right) \bar{v}(\bar{p})\gamma_\mu u(p) - \bar{u}(k)\gamma^\mu u(p) \left(\frac{-i}{t}\right) \bar{v}(\bar{p})\gamma_\mu v(\bar{k}) \Big\}$$

$$\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{e^4}{4}\sum_{\text{spin}} \Big| \frac{1}{s}\bar{u}(k)\gamma^\mu v(\bar{k})\, \bar{v}(\bar{p})\gamma_\mu u(p) - \frac{1}{t}\, \bar{u}(k)\gamma^\mu u(p)\, \bar{v}(\bar{p})\gamma_\mu v(\bar{k}) \Big|^2$$

$$= \frac{e^4}{4}\Big\{ \frac{1}{s^2}\,\mathrm{tr}[\not{k}\gamma^\mu \bar{\not{k}}\gamma^\lambda]\,\mathrm{tr}[\bar{\not{p}}\gamma_\mu \not{p}\gamma_\lambda] + \frac{1}{t^2}\,\mathrm{tr}[\not{k}\gamma^\mu \not{p}\gamma^\lambda]\,\mathrm{tr}[\bar{\not{p}}\gamma_\mu \bar{\not{k}}\gamma_\lambda]$$
$$- \frac{1}{st}\,\mathrm{tr}[\not{k}\gamma^\mu \bar{\not{k}}\gamma_\lambda \bar{\not{p}}\gamma_\mu \not{p}\gamma^\lambda]$$
$$- \frac{1}{st}\,\mathrm{tr}[\not{k}\gamma^\mu \not{p}\gamma_\lambda \bar{\not{p}}\gamma_\mu \bar{\not{k}}\gamma^\lambda] \Big\}$$

now

$$\mathrm{tr}[\not{k}\gamma^\mu \bar{\not{k}}\gamma^\lambda]\ \mathrm{tr}[\bar{\not{p}}\gamma_\mu \not{p}\gamma_\lambda]$$

$$= 4\cdot 4\ [k^\mu \bar{k}^\lambda + k^\lambda \bar{k}^\mu - k\cdot\bar{k}\, g^{\mu\lambda}]\ [\bar{p}_\mu p_\lambda + \bar{p}_\lambda p_\mu - p\cdot\bar{p}\, g_{\mu\lambda}]$$

$$= 16\ [2\, k\cdot p\ \bar{k}\cdot\bar{p} + 2\, k\cdot\bar{p}\ \bar{k}\cdot p - \cancel{2\, k\cdot\bar{k}\ p\cdot\bar{p}\cdot 2} + \cancel{4\, k\cdot\bar{k}\ p\cdot\bar{p}}]$$

$$= 32\ [k\cdot p\ \bar{k}\cdot\bar{p} + k\cdot\bar{p}\ k\cdot p]$$

$$= 8\ [t^2 + u^2]$$

$t = -2k\cdot p = -2\bar{k}\cdot\bar{p}$

$u = -2k\cdot\bar{p} = -2\bar{k}\cdot p$

$$\mathrm{tr}[\not{k}\gamma^\mu \not{p}\gamma^\lambda]\ \mathrm{tr}[\bar{\not{p}}\gamma_\mu \bar{\not{k}}\gamma_\lambda]$$

$s = 2p\cdot\bar{p} = 2k\cdot\bar{k}$

$$= 32\ [k\cdot\bar{p}\ p\cdot\bar{k} + k\cdot\bar{k}\ p\cdot\bar{p}]$$

$$= 8\ [u^2 + s^2]$$

$$\mathrm{tr}[\not{k}\gamma^\mu \bar{\not{k}}\gamma_\lambda \bar{\not{p}}\gamma_\mu \not{p}\gamma^\lambda] = \mathrm{tr}[\gamma^\mu \not{p}\gamma_\lambda \bar{\not{p}}\gamma_\mu \bar{\not{k}}\gamma^\lambda \not{k}]$$

(reverse order & interchange $(\mu \leftrightarrow \lambda)$)

$$= \mathrm{tr}[(-2)\, \not{k}\gamma^\mu \bar{\not{k}} \not{p}\gamma_\mu \bar{\not{p}}]$$

$$= -2\cdot 4\ \mathrm{tr}\ \not{k}\ \bar{k}\cdot p\ \bar{\not{p}}$$

$$= -2\cdot 4\cdot 4\ \bar{k}\cdot p\ k\cdot\bar{p}$$

$$= -8\cdot u^2$$

so

$$\frac{1}{4}\sum_{\text{spin}} |\mathcal{M}|^2 = \frac{e^4}{4} \cdot 8 \left\{ \frac{u^2+t^2}{s^2} + \frac{u^2+s^2}{t^2} + 2\,\frac{u^2}{st} \right\}$$

$$= 2e^4 \left\{ u^2\left(\frac{1}{s}+\frac{1}{t}\right)^2 + t^2\frac{1}{s^2} + s^2\frac{1}{t^2} \right\}$$

$$\frac{d\sigma}{d\cos\theta} = \frac{1}{2s}\,\frac{1}{16\pi}\,2e^4\{\ \}$$

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{s}\left[u^2\left(\frac{1}{s}+\frac{1}{t}\right)^2 + \left(\frac{t}{s}\right)^2 + \left(\frac{s}{t}\right)^2 \right]$$

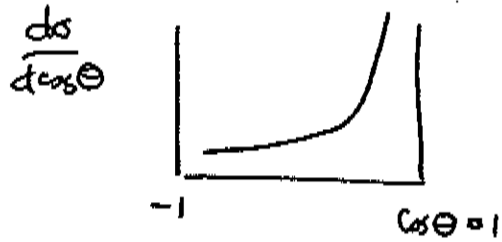
now $s = 4E^2 \quad t = -2E^2(1-\cos\theta) \quad u = -2E^2(1+\cos\theta)$

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{4E^2}\left[(1+\cos\theta)^2\left[\frac{1}{2} - \frac{1}{(1-\cos\theta)}\right]^2 + \left[\frac{(1-\cos\theta)}{2}\right]^2 + \left(\frac{2}{1-\cos\theta}\right)^2 \right]$$

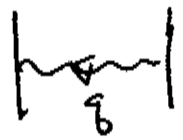
$$= \frac{\pi\alpha^2}{4E^2}\left\{ \frac{4+(1+\cos\theta)^2}{(1-\cos\theta)^2} - \frac{(1+\cos\theta)^2}{(1-\cos\theta)} + \frac{1}{4}\cdot(2+2\cos^2\theta) \right\}$$

$$= \frac{\pi\alpha^2}{4E^2}\left\{ \frac{5+2\cos\theta+\cos^2\theta}{(1-\cos\theta)^2} - \frac{(1+\cos\theta)^2}{(1-\cos\theta)} + \frac{1+\cos^2\theta}{2} \right\}$$

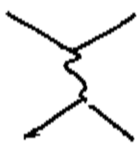
this cross section has a strong singularity as $\theta \to 0$



due to the massless vector exchange in the t-channel



4.) a.) Go back to p.7:



$$= \quad ie^2 \frac{1}{s} \, \bar{u}(k)\gamma^\mu v(\bar{k}) \; \bar{v}(\bar{p})\gamma_\mu u(p)$$

$$= \quad -ie^2 \frac{1}{t} \, \bar{u}(k)\gamma^\mu u(p) \; \bar{v}(\bar{p})\gamma_\mu v(\bar{k})$$

Now

$$\bar{u}(k)\gamma^\mu u(p) = u^\dagger(k)\gamma^0\gamma^\mu u(p) = u^\dagger(k)\begin{pmatrix} \bar{\sigma}^\mu & 0 \\ 0 & \sigma^\mu \end{pmatrix} u(p)$$

so this $= 0$ unless both $e^-(p)$ and $e^-(k)$ are L or R

$$\bar{u}(k)\gamma^\mu v(\bar{k}) = u^\dagger(k)\begin{pmatrix} \bar{\sigma}^\mu & 0 \\ 0 & \sigma^\mu \end{pmatrix} v(\bar{k})$$

this $= 0$ unless $e^-(k)$ is L, $e^+(\bar{k})$ is R or vice versa

so

 is nonzero for

$e_L^- e_R^+ \to e_L^- e_R^+$
$e_L^- e_R^+ \to e_R^- e_L^+$
$e_R^- e_L^+ \to e_L^- e_R^+$
$e_R^- e_L^+ \to e_R^- e_L^+$ only

 is nonzero for

$e_L^- e_L^+ \to e_L^- e_L^+$
$e_L^- e_R^+ \to e_L^- e_R^+$
$e_R^- e_L^+ \to e_R^- e_L^+$
$e_R^- e_R^+ \to e_R^- e_R^+$ only

so

$\left.\begin{array}{l} e_L^- e_R^+ \to e_R^- e_L^+ \\ e_R^- e_L^+ \to e_L^- e_R^+ \end{array}\right\}$ are given by  only

$\left.\begin{array}{l} e_L^- e_L^+ \to e_L^- e_L^+ \\ e_R^- e_R^+ \to e_R^- e_R^+ \end{array}\right\}$ are given by  only

$\left.\begin{array}{l} e_L^- e_R^+ \to e_L^- e_R^+ \\ e_R^- e_L^+ \to e_R^- e_L^+ \end{array}\right\}$ are given by  + 

Now compute the nonzero spinor products using the explicit spinors in the problem set:

$$\bar{v}(\bar{p})\gamma^\mu u(p) = (e_L^- e_R^+) \quad 2E \cdot (-1\ 0)\overbrace{(1,-\vec{\sigma})^\mu}^{\bar{\sigma}^\mu}\begin{pmatrix}0\\1\end{pmatrix}$$
$$= 2E\,(0,1,-i,0)^\mu$$
$$= (e_R^- e_L^+) \quad 2E \quad (0\ -1)\,(1,\vec{\sigma})^\mu\begin{pmatrix}1\\0\end{pmatrix}$$
$$= -2E\,(0\ 1,i,0)^\mu$$

$$\bar{u}(k)\gamma^\mu v(\bar{k}) = (e^-_L e^+_R) \quad 2E\,(-s_2\, c_2)(1,-\vec{\sigma})\begin{pmatrix}-c_2\\-s_2\end{pmatrix}$$

$$= 2E\,(0,\ c_2^2-s_2^2,\ i(c_2^2+s_2^2),\ -2c_2s_2)$$

$$= 2E\,(0,\ \cos\theta,\ i,\ -\sin\theta)$$

$$= (e^-_R e^+_L) \quad 2E\,(c_2\, s_2)(1,\vec{\sigma})\begin{pmatrix}s_2\\-c_2\end{pmatrix}$$

$$= -2E\,(0,\ c_2^2-s_2^2,\ -i(c_2^2+s_2^2),\ -2c_2s_2)$$

$$= -2E\,(0,\cos\theta,\ -i,\ -\sin\theta)$$

$$\bar{u}(k)\gamma^\mu u(p) = (e^-_L\to e^-_L) \quad 2E\,(-s_2c_2)(1,-\vec{\sigma})\begin{pmatrix}0\\1\end{pmatrix}$$

$$= 2E\,(+c_2,\ +s_2,\ -is_2,\ c_2)$$

$$(e^-_R\to e^-_R) \quad 2E\,(c_2\, s_2)(1,\vec{\sigma})\begin{pmatrix}1\\0\end{pmatrix}$$

$$= 2E\,(c_2,\ s_2,\ is_2,\ c_2)$$

$$\bar{v}(\bar{p})\gamma^\mu v(\bar{k}) = (e^+_L\to e^+_L) \quad 2E\,(0\ -1)(1,+\vec{\sigma})\begin{pmatrix}s_2\\-c_2\end{pmatrix}$$

$$= 2E\,(c_2,\ -s_2,\ -is_2,\ -c_2)$$

$$(e^+_R\to e^+_R) \quad 2E\,(-1\ 0)(1,-\vec{\sigma})\begin{pmatrix}-c_2\\-s_2\end{pmatrix}$$

$$= 2E\,(c_2,\ -s_2,\ +is_2,\ -c_2)$$

once again

	$e_L^- e_R^+$	$e_R^- e_L^+$
$\bar{v}(\bar{p})\, \gamma^\mu u(p)$	$2E\,(0\ 1\ -i\ 0)$	$-2E\,(0\ 1\ i\ 0)$
$\bar{u}(k)\, \gamma^\mu v(\bar{k})$	$2E\,(0\ \cos\theta\ +i\ -\sin\theta)$	$-2E\,(0\ \cos\theta\ -i\ -\sin\theta)$
	$e_L^- \to e_L^-$	$e_R^- \to e_R^-$
$\bar{u}(k)\, \gamma^\mu u(p)$	$2E\,(c_2\ s_2\ -is_2\ c_2)$	$2E\,(c_2\ s_2\ is_2\ c_2)$
$\bar{v}(\bar{p})\, \gamma^\mu v(\bar{k})$	$2E\,(c_2\ -s_2\ -is_2\ -c_2)$	$2E\,(c_2\ -s_2\ +is_2\ -c_2)$
	$e_L^+ \to e_L^+$	$e_R^+ \to e_R^+$

then: cbt w. Lorentz product

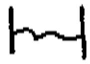
$$= \frac{ie^2}{s}\cdot(2E)^2 \begin{cases} -(\cos\theta+1) & e_L^- e_R^+ \to e_L^- e_R^+ \\ +(\cos\theta-1) & e_L^- e_R^+ \to e_R^- e_L^+ \\ +(\cos\theta-1) & e_R^- e_L^+ \to e_L^- e_R^+ \\ -(\cos\theta+1) & e_R^- e_L^+ \to e_R^- e_L^+ \end{cases}$$

$$= -\frac{ie^2}{t}(2E)^2 \begin{cases} 2c_2^2 + 2s_2^2 = 2 & e_L^- e_L^+ \to e_L^- e_L^+ \\ 2c_2^2 = (1+\cos\theta) & e_L^- e_R^+ \to e_L^- e_R^+ \\ 2c_2^2 = (1+\cos\theta) & e_R^- e_L^+ \to e_R^- e_L^+ \\ 2c_2^2 + 2s_2^2 = 2 & e_R^- e_R^+ \to e_R^- e_R^+ \end{cases}$$

& recognize

$$s = 4E^2$$
$$t = -2E^2(1-\cos\theta)$$
$$u = -2E^2(1+\cos\theta)$$

so

$e^-_L e^+_L \to e^-_R e^+_R$  $= -2ie^2 \frac{s}{t}$

$e^-_R e^+_L \to e^-_R e^+_L$

$e^-_L e^+_R \to e^-_R e^+_L$  $= +2ie^2 \frac{t}{s}$

$e^-_R e^+_L \to e^-_L e^+_R$

$e^-_L e^+_R \to e^-_R e^+_R$  $+$  $= -2ie^2 u \left(\frac{1}{s} + \frac{1}{t}\right)$

$e^-_R e^+_L \to e^-_R e^+_L$

then the spin-averaged $\frac{d\sigma}{d\cos\theta}$ is

$$\frac{d\sigma}{d\cos\theta} = \frac{1}{2s} \cdot \frac{1}{16\pi} \cdot \frac{1}{4} \underbrace{\sum |m|^2}_{4e^4 \cdot 2 \cdot \left[\left(\frac{s}{t}\right)^2 + \left(\frac{t}{s}\right)^2 + u^2\left(\frac{1}{s}+\frac{1}{t}\right)^2\right]}$$

$$= \frac{\pi \alpha^2}{s} \left[\left(\frac{s}{t}\right)^2 + \left(\frac{t}{s}\right)^2 + u^2\left(\frac{1}{s} + \frac{1}{t}\right)^2 \right]$$