Physics 330 – Problem Set # 6

(due Wednesday, November 9)

1. Peskin and Schroeder, problem 4.4.

2. Peskin and Schroeder, problem 5.1.

3. Peskin and Schroeder, problem 5.2.

4. In our discussion of $e^+e^- \to \mu^+\mu^-$ in class, we saw that the limit $m_\mu \to 0$ of the cross section was built up in a simple way as a sum of squares of amplitudes for scattering processes with fermions of definite helicity in the initial and final states. We used the following explicit spinors to compute the amplitudes:

$$\begin{aligned}
e^- \text{ with } \vec{p} \parallel \hat{3} \ : \qquad & u_R(p) = \sqrt{2E}\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad u_L(p) = \sqrt{2E}\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \\
e^- \text{ in } \theta \text{ direction} : \qquad & u_R(p) = \sqrt{2E}\begin{pmatrix} 0 \\ 0 \\ c_2 \\ s_2 \end{pmatrix} \quad u_L(p) = \sqrt{2E}\begin{pmatrix} -s_2 \\ c_2 \\ 0 \\ 0 \end{pmatrix} \\
e^+ \text{ with } \vec{p} \parallel -\hat{3} \ : \qquad & v_R(p) = \sqrt{2E}\begin{pmatrix} -1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad v_L(p) = \sqrt{2E}\begin{pmatrix} 0 \\ 0 \\ 0 \\ -1 \end{pmatrix} \\
e^+ \text{ opposite to } \theta : \qquad & v_R(p) = \sqrt{2E}\begin{pmatrix} -c_2 \\ -s_2 \\ 0 \\ 0 \end{pmatrix} \quad v_L(p) = \sqrt{2E}\begin{pmatrix} 0 \\ 0 \\ s_2 \\ -c_2 \end{pmatrix} \qquad (1)
\end{aligned}$$

where $c_2 = \cos\theta/2$, $s_2 = \sin\theta/2$, L, R refer to the physical spin direction of the particle or antiparticle, and for antiparticles I have given $v(p)$, not $\overline{v}(p)$. Use these explicit spinors to redo the calculation of the Bhabha scattering cross section.

   (a) There are $2^4 = 16$ possible helicity amplitudes. Show that 10 of these are zero, 4 involve only one Feynman diagram, and 2 require a sum of two Feynman diagrams.

   (b) Compute the nonzero helicity amplitudes. Show that the squares of these amplitudes are equal in pairs. (This is a consequence of C or P invariance.)

   (c) Show that the sum of the three contributions reproduces the result of problem 3.