# Physics 330 – Final Exam

This exam is due at noon on Friday, December 16. Please hand it in to Alex Giryavets in Varian 361. If you have any questions about the exam, please contact me at mpeskin@slac.stanford.edu or 926-3250. If errata are reported, I will announce them on the course Web page.

Please do not collaborate on this exam. Please return the exam in a blue book (or multiple blue books) with the honor code acknowledgement signed. Alex Giryavets has blue books, if you need some.

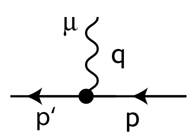
The exam is worth a total of 100 points. The distribution of points is indicated below.

This problem has some historical interest, because it concerns a very interesting exercise in quantum field theory that Feynman solved under circumstances that clarified for him the power and general applicability of the methods taught in this course. I attach the story at the end of the exam.

Since the problem is of historical interest only, I doubt that you will find a clear treatment of it in the literature. I recommend that you simply try to answer the questions using methods from Physics 330. However, the exam is open-book. If you find a useful reference other than the class textbook and notes, feel free to use it, as long as you cite the reference in your solution.

This problem concerns a simple field theory description of the interactions of pions and nucleons.

a. (10 points) To begin, we will need to know something about the electromagnetic interactions of scalar particles. I claim that, if $\phi$ is a scalar field of charge $(-e)$, like the electron, then the Feynman rule for the coupling of $\phi$ to a photon is :




$$= -ie(p + p')^\mu \tag{1}$$

This interaction can be derived from the Lagrangian

$$\mathcal{L} = (D_\mu \phi)^* D^\mu \phi - m^2 \phi^* \phi \tag{2}$$

with $D_\mu = \partial_\mu + ieA_\mu$. The term linear in $A_\mu$ is

$$\delta\mathcal{L} = \partial_\mu \phi^* (ieA^\mu)\phi - ieA^\mu \phi^* \partial_\mu \phi \tag{3}$$

Since the incoming $\phi$ has a wavefunction $e^{-ip\cdot x}$, we can replace $\partial_\mu \phi$ by $(-ip_\mu)$, and treat $\partial_\mu \phi^*$ similarly, to obtain the Feynman rule.

Now, here is the question: Verify that this vertex satisfies the following consistency checks:

(1) Show that the nonrelativistic electromagnetic scattering of a charged scalar particle $\phi$ from an electron gives a Coulomb potential with the correct sign. Show this also for the positively charged antiparticle $\phi^*$.

(2) Show that the vertex $\gamma \to \phi\phi^*$ for pair production, which would be used to compute $e^+e^- \to \phi\phi^*$, satisfies current conservation for $\phi$, $\phi^*$ on-shell.

b. (10 points) Compute the cross section for $e^+e^- \to \phi\phi^*$, averaged over electron spins. You may ignore the mass of the electron, but not the mass of the $\phi$. How does the total cross section compare with that for $e^+e^- \to \mu^+\mu^-$ as $E_{cm} \to \infty$?

c. (5 points) Now turn to the description of pion-nucleon interactions. Let $N$ represent a doublet of Dirac fermion fields

$$N = \begin{pmatrix} p \\ n \end{pmatrix} \tag{4}$$

where $p$, $n$ represent the Dirac fermion fields of the proton and neutron. Let $\pi^i$, $i = 1, 2, 3$ represent three pseudoscalar pion fields

$$\pi^0 = \pi^3 \qquad \pi^{\pm} = \frac{1}{\sqrt{2}}(\pi^1 \mp i\pi^2) \tag{5}$$

A coupling of the pions to the nucleons can be written

$$\Delta\mathcal{H} = ig_{\pi NN}\pi^i \overline{N}\gamma^5\sigma^i N \tag{6}$$

The notation is slightly tricky; the $\gamma^5$ acts on the Dirac indices to create a pseudoscalar coupling. The $\sigma^i$ acts on the 2-component indices that the distinguish the $p$ and the $n$. The $\pi^-$ field *destroys* a $\pi^-$ and *creates* a $\pi^+$. Work out the Feynman rules. Write explicitly the vertex by which a proton emits a $\pi^+$ and converts to a neutron. Show that the electric charge assignments work out correctly for this process.

For simplicity, let the three pions have equal masses $m_\pi$ and let the proton and neutron have equal masses $m_N$.

d. (10 points) Consider proton-proton scattering in the leading order of pertubation theory. Work out the leading term in the scattering amplitude in the limit of nonrelativistic velocities. Show that this term represents a spin-spin interaction, mediated by a potential related to the Yukawa potential. Write the similar expressions for neutron-neutron and proton-neutron scattering.

Since we do not obtain a universally attractive potential, the interaction (6) does not, by itself, explain the binding of protons and neutrons into nuclei. For this, we need a more complicated model, such as the linear sigma model considered in Problem Set 5.

e. (10 points) The coupling (6) is called the 'pseudoscalar' interaction. It is also possible to write a 'pseudovector' interaction:

$$\Delta\mathcal{H} = -\frac{g_{\pi NN}}{2m_N}\partial_\mu\pi^i \overline{N}\gamma^\mu\gamma^5\sigma^i N \tag{7}$$

Work out the leading term for nonrelativistic proton-proton scattering in the pseudovector model, and show that it is identical to the result for the pseudoscalar model.

f. (10 points) Write the full relativistic amplitudes for proton-proton scattering in the leading order of perturbation theory in the pseudoscalar and pseudovector theories. Show that these are identical. Does the trick work also for other scattering processes, considered in leading order?

g. (5 points) In classical physics, one might not expect the neutron to interact with electromagnetic fields. However, in quantum field theory, the neutron can fluctuate to a proton plus a pion. Thus, there are nonzero contributions to the electromagnetic form factors of the neutron at one-loop order. Draw the diagrams that contribution to the $F_1$ and $F_2$ form factors of the neutron in order $g^2_{\pi NN}$.

h. (10 points) Compute these diagrams, up to the integrals over Feynman parameters. Use dimensional regularization. Show that the results for $F_1$ and $F_2$ are finite both in the UV and in the IR.

i. (10 points) Show explicitly that $F_1(0) = 0$, as required by general principles.

j. (10 points) The charge radius $r$ of the neutron can be defined by

$$F_1(q^2) = 6q^2r^2 + \mathcal{O}(q^4) \tag{8}$$

Compute $r^2$ in terms of the parameters of this theory.

k. (10 points) Show that the pseudovector theory gives a different expression for the form factors of the neutron from that of the pseudoscalar theory, one which is UV divergent. Is it only $F_1$ or also $F_2$ that is divergent?

Self-evaluation: To record a satisfactory performance on this exam, please complete at least through part (g). Prospective theorists should slog through to the end.

Finally, here is the story about Feynman. It is one of my favorite Feynman stories, not least because probably happened more or less as it is told. I quote extensively from Jagdish Mehra's biography, *The Beat of a Different Drum*, Oxford U. Press, 1994:

At the 1949 APS meeting, Murray Slotnik presented new results, which he had obtained after two years of calculation, concerning the interaction between an electron and a nuetron. He had found that the answers for the pseudoscalar theory and the pesudovector theory were different. In fact, in the case of the pseudovector theory, the answer was logarithmically divergent, but, in the pseudoscalar theory, it was convergent and gave well-defined answers. [J. Robert] Oppenheimer, who was in the audience, asked Slotnik: 'Well, what about Case's theorem?' By this he meant the new result of Kenneth Case (who, at that time, was a postdoctoral fellow at Institute for Advanced Study in Princeton), which was going to be reported the next day. [Oppenheimer was, at that time, the Director of the Institute.] Case had announced that he had proved that the results for both the pseudoscalar and pseudovector theories were the same. Slotnick answered: 'I never heard of Case's theorem!'

Feynman had missed Slotnick's talk, but somebody asked him about the discussion between Oppenheimer and Slotnick. He went to Slotnick and said: 'Look, I am very anxious to try out if I understand what these things mean. So just tell me what you did.' He replied, 'I scattered the electron off the neutron and I have a correction due to the mesons.'

Feynman described what happened next as follows: 'This was a welcome opportunity to test my guesses as to whether I really understood what these two couplings were. So I went home, and during the evening I worked out the electron-neutron scattering for the pseudoscalar and pseudovector couplings, saw that they were not equal and subtracted them, and worked out the difference in detail. The next day, at the meeting, I saw Slotnick and said, "Slotnick, I worked it out last night, I wanted to see if I got the same answers you do. I got a different answer for each coupling—but I would like to check in detail with you because I want to be sure of my methods." And he said, "What do you mean you worked it out last night? It took me six months!" And, when we compared the answers, he looked at mine and he asked, "What is that $Q$ in there, that variable $Q$?" ... I said, "That's the momentum transferred by the electron, the electron deflected by different angles." "Oh," he said, "No, I only have the limiting value as $Q$ approaches [0 for] the forward scattering." Well, it was easy enough to just substitute $Q$ equals zero in my answers as he did. But it took him six months to do the case of zero momentum transfer, whereas, during one evening, I had done the finite and arbitrary momentum transfer. That was a thrilling moment for me, like receiving the Nobel Prize, because that convinced me, at last, I did have some kind of method and technique and understood how to do something that other people did not know how to do. That was my moment of triumph in which I realized I really had succeeeded in working out something worthwhile.' ...

As Feynman further recalled, next day Case reported his theorem at the APS meeting. 'And, just to be annoying, when Case finished, I said, "Yeah, but what about Slotnick's calculation?" You know, I mean Oppenheimer was imperious. If Case proved the theorem, it must be true. And I argued, "What about Slotnick's calculation? That theorem can't be true." And everybody laughed becuase it was perfectly logical to suppose the theorem is at fault rather than the calculation, you know. So [Oppenheimer] said: "Well, maybe Slotnick's calculation is wrong." I said, "No, I checked it last night and it's all right. I believe it's right." '