

## Physics 330 – Final Exam

This exam is due at noon on Thursday, December 9. Please hand it in to Alex Giryavets in Varian 361. If you have any questions about the exam, please contact me at mpe-skin@slac.stanford.edu or 926-3250. If errata are reported, I will announce them on the course Web page.

Please do not collaborate on this exam. Please return the exam in a blue book (or multiple blue books) with the honor code acknowledgement signed. Alex Giryavets has blue books, if you need one.

The exam is worth a total of 100 points. The distribution of points is indicated below.

The problem deals with some aspects of the phenomenology of the Higgs boson. Even though the Higgs boson has not yet been discovered, there is a large literature on its properties. Fortunately, most of this literature is too sophisticated to be useful to you for the purpose of solving this exam. I recommend that you simply try to answer the questions using methods from Physics 330. However, the exam is open-book. If you find a useful reference other than the class textbook and notes, feel free to use it, as long as you cite the reference in your solution.

- a. (20 points) Let  $f$  be the Dirac field representing a quark or lepton. The Higgs boson is a scalar particle, associated with a field  $h$  that couples to each fermion field according to

$$\Delta H = \frac{m_f}{v} \bar{f} f h \quad (1)$$

where  $m_f$  is the fermion mass and  $v = 246$  GeV. Draw the Feynman diagram for Higgs decay into  $f\bar{f}$ . Compute the partial width using the formula

$$\Gamma(h \rightarrow f\bar{f}) = \frac{1}{2m_h} \int d\Pi |\mathcal{M}(h \rightarrow f\bar{f})|^2 \quad (2)$$

- b. (5 points) Taking the masses of the heaviest quarks and leptons to be:

$$\begin{aligned} \text{charge } \frac{2}{3} \text{ quarks} &: m_c = 1.2 \text{ GeV} & m_t = 178 \text{ GeV} \\ \text{charge } -\frac{1}{3} \text{ quarks} &: m_b = 4.2 \text{ GeV} \\ \text{charge } -1 \text{ leptons} &: m_\mu = 0.105 \text{ GeV} & m_\tau = 1.77 \text{ GeV} \end{aligned} \quad (3)$$

compute the total width and decay rate of a Higgs boson of mass 120 GeV and its branching fractions to each of these species. Do not forget the color factor of 3. (The Higgs boson is also supposed to decay to  $W$  and  $Z$  bosons; you should ignore those contributions in this exam.)

- c. (20 points) Higgs bosons can be produced in quark-antiquark (or proton-antiproton) collisions through the process

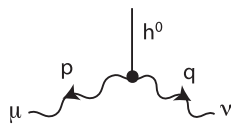
$$q + \bar{q} \rightarrow h + \gamma \quad (4)$$

This process is not very much discussed in the literature because it is not a very effective way to produce Higgs bosons. Nevertheless, compute the appropriately polarization summed and averaged differential cross section for this process in the CM frame. You may ignore the mass of the quark, except, of course, in the coupling in eq. (1). Take care with the color factor: Assume that quarks and antiquarks of all three colors are equally probable in the proton wavefunction. Express the cross section as

$$\frac{d^2\sigma}{d^2p_\perp} \quad (5)$$

where  $\vec{p}_\perp$  is the photon momentum perpendicular to the collision axis. This expression is invariant to longitudinal boosts. Notice that the cross section depends on the quark species and is much larger for heavier quarks.

- d. (5 points) Assuming that the proton contains a  $b$  quark or antiquark with probability about 0.1, estimate the order of magnitude of the cross section for production of a Higgs boson in association with a photon with  $p_\perp > 100$  GeV. The momentum of the  $b$  quark is determined by the proton wavefunction. It might be anything from near 0 to about 1% of the proton energy, so just choose the value of  $s$  to maximize the cross section. You do not have to do a detailed calculation; I just want the order of magnitude. If the Large Hadron Collider at CERN will produce enough proton-proton collisions per year to give 100 events of a process whose cross section is 1 fb ( $1 \times 10^{-39}$  cm<sup>2</sup>), roughly how many events of this type will be produced? (We will discuss the quark structure of the proton in some detail in Physics 331; then you will know how to get a more precise prediction.)
- e. (5 points) If  $m_f$  is small, the cross section computed in part (c) is absurdly small. However, we could obtain a larger answer if the Higgs boson can couple directly to 2 photons. This vertex is generated by 1-loop Feynman diagrams. It takes the form:



$$= -i \frac{\alpha}{4\pi} C (g^{\mu\nu} p \cdot q - p^\nu q^\mu) \quad (6)$$

Show that this vertex satisfies the Ward Identity.

- f. (15 points) Using this vertex, draw a new Feynman diagram for the process  $q\bar{q} \rightarrow h\gamma$ . Assuming that  $m_f$  is so small that the previous diagrams can be neglected, compute the differential cross section for  $h\gamma$  production. Your answer should be proportional to  $|C|^2$ .
- g. (20 points) Draw the Feynman diagrams by which a loop containing a heavy quark generates the vertex in part (e). For definiteness, use the heavy  $t$  quark. Compute these diagrams in the limit  $m_t^2 \gg p \cdot q, m_h^2$ ; that is, ignore all terms that vanish in the

limit  $m_t \rightarrow \infty$ . Use dimensional regularization. You will find a peculiar feature: A crucial term will be proportional to  $(d - 4)$  times a divergent diagram. However, this term will give a finite and definite value in dimensional regularization, since

$$\lim_{d \rightarrow 4} (d - 4)\Gamma(2 - d/2) = -2 . \quad (7)$$

(In Physics 332, if you are still around, I will explain what is going on here and why it is actually correct.) Show that the amplitude actually takes the form of eq. (6). Find the explicit expression for  $C$ . (Again, there is a second contribution from  $W$  bosons in the loop. Please ignore it for the purpose of this exam.)

- h. (10 points) Compute the partial width of the Higgs boson to  $\gamma\gamma$ . Using the results of parts (b) and (g), obtain a numerical answer for the branching ratio of the Higgs to  $\gamma\gamma$ .

Self-evaluation: To record a satisfactory performance on this exam, please complete at least through part (f). Prospective theorists should slog through to the end.