Physics 330 - Final Exam

Solutions

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a.)

(1)  $= (-ie)\, \bar{u}(p')\gamma^\mu u(p) \; \frac{-i g_{\mu\nu}}{q^2} \; (-ie)(k+k')^\nu$

$$i\mathcal{M} = ie^2 \; \bar{u}(p')\gamma^\mu u(p) \; (k+k')_\mu \; \frac{1}{q^2}$$

In the limit of nonrelativistic scattering

$$\bar{u}^{(s')}(p')\gamma^\mu u^{(s)}(p) \cong \big(\underset{\mu=0}{2m_e\, \xi^{s'\dagger}\xi^s} \, , \; \underset{\mu=i}{\vec{0}}\big)$$

$$(k+k')^\mu \cong (2m_\phi \, , \vec{0})$$

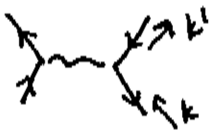
$$q^2 = -|\vec{q}|^2$$

$$i\mathcal{M} = -i\frac{e^2}{|\vec{q}|^2} \, \delta^{s's} \, 2m_e \, 2m_\phi$$

which corresponds to the Born approximation with

$$\tilde{V}(\vec{q}) = +\frac{e^2}{|\vec{q}|} \qquad V(r) = +\frac{e^2}{4\pi r} \qquad \underline{\text{repulsive}}$$

for $e^- \phi^*$ scattering



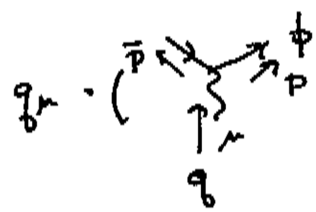
$$i\mathcal{M} = -ie\,\bar{u}\gamma^\mu u \left(\frac{-ig_{\mu\nu}}{q^2}\right)(-ie)(-k-k')^\nu$$

$$= (-1)\cdot \text{previous}$$

so

$$i\mathcal{M} = +i\frac{e^2}{|\vec{q}|^2}\,\delta^{s's}\,2m_e\,2m_\phi \quad \Rightarrow \quad V(r) = -\frac{e^2}{4\pi r}$$

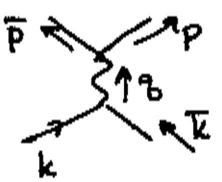
attractive !

(2.) $q_\mu \cdot \Big($  $\Big) = -ie\,(p-\bar{p})\cdot q$

In this kinematics $q = p+\bar{p}$ so the above

$$= -ie\,(p-\bar{p})\cdot(p+\bar{p}) = -ie\,(p^2-\bar{p}^2) = -ie\,(m_\phi^2 - m_\phi^2)$$

$$= 0 \;\checkmark$$

b.) 

$$i\mathcal{M} = -ie\;\bar{v}(\bar{k})\gamma^\mu u(k)\cdot\frac{-i}{q^2}\cdot -ie\,(p-\bar{p})_\mu$$

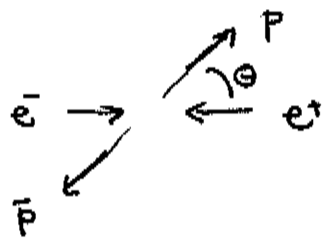
$$= \frac{ie^2}{q^2}\;\bar{v}(\bar{k})\gamma^\mu u(k)\;(p-\bar{p})_\mu$$

$$\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{1}{4}\frac{e^4}{q^4}\, \mathrm{tr}[\not{\bar{k}}\gamma^\mu \not{k}\gamma^\nu]\, (p-\bar{p})_\mu (p-\bar{p})_\nu$$

$$= \frac{e^4}{q^4}\left(\bar{k}^\mu k^\nu + \bar{k}^\nu k^\mu - g^{\mu\nu}\, k\cdot\bar{k}\right)(p-\bar{p})_\mu (p-\bar{p})_\nu$$

$$= \frac{e^4}{q^4}\left[2\, k\cdot(p-\bar{p})\ \bar{k}\cdot(p-\bar{p}) - (p-\bar{p})^2\, k\cdot\bar{k}\right]$$

In the kinematics



$$k = (E,0,0,E) \qquad\qquad p = (E,\ p\sin\theta,\ 0\ \ p\cos\theta)$$

$$\bar{k} = (E,0,0,-E) \qquad\qquad \bar{p} = (E,-p\sin\theta,\ 0,-p\cos\theta)$$

$$p-\bar{p} = 2p\,(0,\ \sin\theta,\ 0,\ \cos\theta)$$

$$k\cdot(p-\bar{p}) = -\,E\cdot 2p\cdot\cos\theta$$

$$\bar{k}\cdot(p-\bar{p}) = +\,E\cdot 2p\cdot\cos\theta$$

$$(p-\bar{p})^2 = -\,4p^2$$

$$k\cdot\bar{k} = 2E^2 \qquad\qquad q^2 = 4E^2$$

$$= \frac{e^4}{(4E^2)^2}\left\{-2\cdot 4E^2p^2\cos^2\theta + 4p^2\cdot 2E^2\right\}$$

$$= \frac{2e^4}{4E^2}\, p^2\,(1-\cos^2\theta) = \frac{1}{2}\, e^4\left(\frac{p}{E}\right)^2 \sin^2\theta$$

$$\frac{d\sigma}{d\cos\theta} = \frac{1}{2s}\,\frac{1}{16\pi}\,\frac{p}{E}\,\frac{1}{4}\sum_{spins}|M|^2$$

$$= \frac{1}{2s}\,\frac{1}{16\pi}\,\frac{1}{2}e^4\left(\frac{p}{E}\right)^3\sin^2\theta$$

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{4s}\left(\frac{p}{E}\right)^3\sin^2\theta$$

the total cross section is :

$$\int_{-1}^{1} d\cos\theta\,\sin^2\theta = \int_{-1}^{1} d\cos\theta\,(1-\cos^2\theta)$$
$$= 2 - 2/3 = 4/3$$

$$\sigma = \frac{\pi\alpha^2}{3s}\left(\frac{p}{E}\right)^3$$

as $E/m \to \infty$ $\quad \frac{p}{E} \to 1$

$$\sigma(e^+e^- \to \phi\phi^*) \to \frac{\pi\alpha^2}{3s}$$

$$\sigma(e^+e^- \to \mu^+\mu^-) \to \frac{4\pi\alpha^2}{3s}$$

so $\quad \sigma(e^+e^- \to \phi\phi^*) \Rightarrow \frac{1}{4}\left(\sigma(e^+e^- \to \mu^+\mu^-)\right)$

c.) Write out $\Delta\mathcal{H}$ in detail:

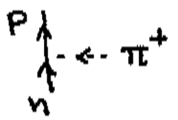
$$\Delta\mathcal{H} = i g_{\pi NN}\, \pi^i\, \bar{N} \gamma^5 \sigma^i N$$

$$= i g_{\pi NN} \left\{ \pi^1 (\bar{p}\,\bar{n}) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \gamma^5 \begin{pmatrix} p \\ n \end{pmatrix} + \pi^2 (\bar{p}\,\bar{n}) \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \gamma^5 \begin{pmatrix} p \\ n \end{pmatrix} \right.$$
$$\left. + \pi^3 (\bar{p}\,\bar{n}) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \gamma^5 \begin{pmatrix} p \\ n \end{pmatrix} \right\}$$

$$= i g_{\pi NN} \left\{ \bar{p}\, (\pi^1 - i\pi^2) \gamma^5 n + \bar{n}\, (\pi^1 + i\pi^2) \gamma^5 p \right.$$
$$\left. + \pi^3 (\bar{p}\gamma^5 p - \bar{n}\gamma^5 n) \right\}$$

$$= i g_{\pi NN} \left\{ \sqrt{2}\, \pi^+ \bar{p}\gamma^5 n + \sqrt{2}\, \pi^- \bar{n}\gamma^5 p \right.$$
$$\left. + \pi^0 (\bar{p}\gamma^5 p - \bar{n}\gamma^5 n) \right\}$$

then the Feynman rules are:



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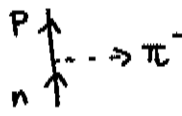
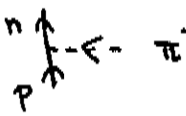


$= g_{\pi NN} \sqrt{2}\, \gamma^5$



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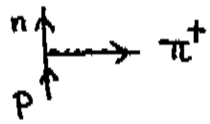
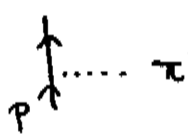


$= g_{\pi NN} \sqrt{2}\, \gamma^5$

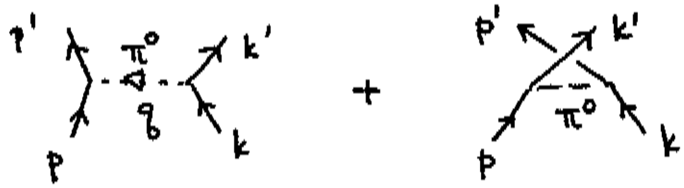


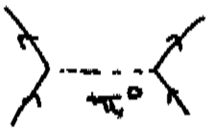
$= g_{\pi NN} \gamma^5$

$= -g_{\pi NN} \gamma^5$

and all charge assignments are consistent.

d.) for proton-proton scattering:





$$i\mathcal{M} = (g_{\pi NN})^2 \, \bar{u}(p')\gamma^5 u(p) \frac{i}{q^2 - m_\pi^2} \bar{u}(k')\gamma^5 u(k)$$

$$u(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\,\xi \\ \sqrt{p\cdot\bar{\sigma}}\,\xi \end{pmatrix} = \begin{pmatrix} \sqrt{m_N - \vec{\sigma}\cdot\vec{p} + O(p^2)}\;\;\xi \\ \sqrt{m_N + \vec{\sigma}\cdot\vec{p} + O(p^2)}\;\;\xi \end{pmatrix}$$

$$= \sqrt{m_N} \begin{pmatrix} (1 - \frac{1}{2}\frac{\vec{\sigma}\cdot\vec{p}}{m_N})\,\xi \\ (1 + \frac{1}{2}\frac{\vec{\sigma}\cdot\vec{p}}{m_N})\,\xi \end{pmatrix} + O(p^2)$$

$$\bar{u}(p')\gamma^5 u(p) = m_N \; \xi'^\dagger \left((1 - \tfrac{1}{2}\tfrac{\vec{\sigma}\cdot\vec{p}\,'}{m_N}), (1 + \tfrac{1}{2}\tfrac{\vec{\sigma}\cdot\vec{p}\,'}{m_N}) \right) \overset{\gamma^0}{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}} \overset{\gamma^5}{\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}}$$

$$\cdot \begin{pmatrix} (1 - \frac{1}{2m_N}\vec{\sigma}\cdot\vec{p}) \\ (1 + \frac{1}{2m_N}\vec{\sigma}\cdot\vec{p}) \end{pmatrix} \xi \quad + O(p^2)$$

$$= m_N \; \xi'^\dagger \Big[\; (1 - \frac{\vec{\sigma}\cdot\vec{p}\,'}{2m_N}) \, (+1) \, (1 + \frac{\vec{\sigma}\cdot\vec{p}}{2m_N})$$

$$+ \; (1 + \frac{\vec{\sigma}\cdot\vec{p}\,'}{2m_N}) \, (-1) \, (1 - \frac{\vec{\sigma}\cdot\vec{p}}{2m_N}) \Big] \; \xi$$

$$= m_N \; \xi'^\dagger \left[- \left(\frac{\vec{\sigma}\cdot \vec{p}\,' - \vec{\sigma}\cdot\vec{p}}{2m_N}\right) \cdot 2 \quad + O(p^2) \right] \xi$$

so

$$\bar{u}(p')\gamma^5 u(p) = -\xi'^{\dagger}\vec{\sigma}\cdot(\vec{p}'-\vec{p})\,\xi = -\xi'^{\dagger}\vec{\sigma}\xi\cdot\vec{q}$$

where $\vec{q} = \vec{p}'-\vec{p}$

Now $\xi'^{\dagger}\vec{\sigma}\xi$ is related to the matrix element of the proton spin

$$\langle\xi'|\,\vec{S}\,|\xi\rangle = \xi'^{\dagger}\left(\frac{\hbar\vec{\sigma}}{2}\right)\xi$$

so

$$\bar{u}(p')\gamma^5 u(p) = -2\,\vec{S}\cdot\vec{q}$$

$$i\mathcal{M} = i\,(2m_N)^2\cdot g_{\pi NN}^2\cdot\frac{\vec{S}_p\cdot\vec{q}}{m_N}\,\frac{1}{q^2-m^2}\left(\frac{-\vec{S}_k\cdot\vec{q}}{m_N}\right)$$

for elastic scattg: $q^\mu = (0,\vec{q})$ $\quad q^2 = -|\vec{q}|^2$

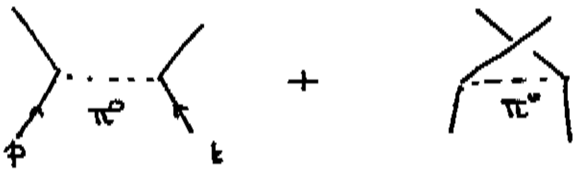
$$i\mathcal{M} = -i\,(2m_N)^2\,g_{\pi NN}^2\,\vec{S}_p\cdot(-i\vec{q})\,\vec{S}_k\cdot(-i\vec{q})\,\frac{1}{|\vec{q}|^2+m_\pi^2}$$

in coordinate space

$$\int\frac{d^3q}{(2\pi)^3}e^{i\vec{q}\cdot\vec{r}}\frac{1}{|\vec{q}|^2+m_\pi^2} = V_Y(\vec{r}) = \frac{e^{-m_\pi r}}{4\pi r}$$ Yukawa potential

$$i\mathcal{M} = -i\,(2m_N)^2\int d^3r\;\frac{\vec{S}_p\cdot\vec{\nabla}}{m_N}\,\frac{\vec{S}_k\cdot\vec{\nabla}}{m_N}\left[g_{\pi NN}^2\,V_Y(\vec{r})\right]$$

that is:

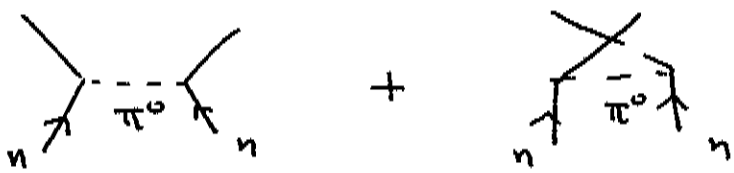


$$iM = -i\,(2m_N)^2 \; S_p^i \, S_k^j \left(\frac{-iq^i}{m_N}\right)\left(\frac{-iq^j}{m_N}\right) \frac{g_{\pi NN}^2}{|\vec{q}|^2 + m_\pi^2}$$

$$- \text{ (exchge term.)}$$

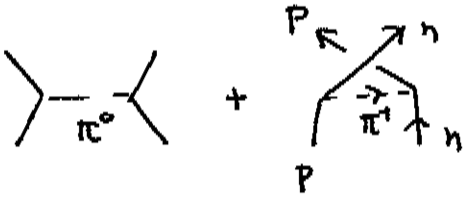
$p' \leftrightarrow k'$, $\xi_p' \leftrightarrow \xi_k'$

for nn scattering we have.



which leads to the same expression

for pn scattering, we have



the direct term is

$$iM = +i\,(2m_N)^2 \; S_p^i \, S_n^j \left(\frac{-iq^i}{m_N}\right)\left(\frac{-iq^j}{m_N}\right) \frac{g_{\pi NN}^2}{|\vec{q}|^2 + m_\pi^2}$$

the exchge term is

$$i\mathcal{M} = (-1)\cdot(-i)\;\xi_{k'}^{\dagger}\frac{\sigma^i}{2}\xi_p\;\xi_{p'}^{\dagger}\frac{\sigma^j}{2}\xi_k\left(\frac{-iq^i}{m_N}\right)\left(\frac{-iq^j}{m_N}\right)\frac{g_{\pi NN}^2}{|\vec{q}|+m_\pi^2}$$

e.) For the pseudovector interaction, if q^μ is the incoming π momentum

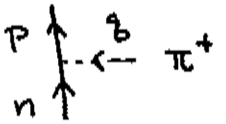
$$\partial_\mu \rightarrow -iq^\mu$$

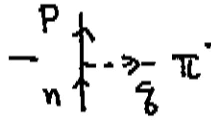
$$\Delta\mathcal{H} = +i\frac{q^\mu}{2m_N}g_{\pi NN}\bar{N}\gamma^\mu\gamma^5\sigma^i N\,\pi^i$$

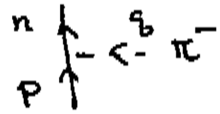
repeating the analysis on p. 5:

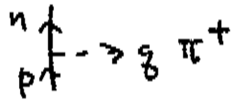
$$= i\frac{q^\mu g_{\pi NN}}{2m_N}\Big\{\sqrt{2}\,\pi^+\,\bar{p}\gamma^\mu\gamma^5 n + \sqrt{2}\,\pi^-\,\bar{n}\gamma^\mu\gamma^5 p$$
$$+\ \pi^0\left(\bar{p}\gamma^\mu\gamma^5 p - \bar{n}\gamma^\mu\gamma^5 n\right)\Big\}$$

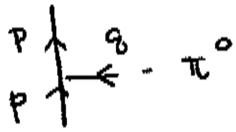
so

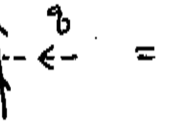


$= g_{\pi NN}\sqrt{2}\,\frac{q_\mu\gamma^\mu\gamma^5}{2m_N} = -$




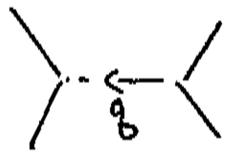
$= g_{\pi NN}\sqrt{2}\,\frac{q_\mu\gamma^\mu\gamma^5}{2m_N} = -$




$= g_{\pi NN}\,\frac{q_\mu\gamma^\mu\gamma^5}{2m_N}$


$= -g_{\pi NN}\,\frac{q_\mu\gamma^\mu\gamma^5}{2m_N}$

f.) For proton proton scattering in the pseudovector model, consider first

 $= \quad g^2_{\pi NN} \quad \dfrac{\bar{u}(p')\, q_\mu \gamma^\mu \gamma^5 u(p)}{2m_N} \; \dfrac{i}{q^2 - m_\pi^2}$

$$\cdot \; \frac{\bar{u}(k')\,[-q_\mu]\gamma^\mu \gamma^5\, u(k)}{2m_N}$$

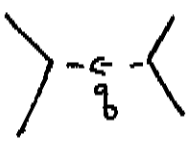
$$\bar{u}(p')\gamma^\mu\gamma^5 u(p) \cong \sqrt{m_N}\,(\xi'^\dagger, \xi'^\dagger)\underbrace{\begin{pmatrix}0&1\\1&0\end{pmatrix}}_{\gamma^0}\begin{pmatrix}0&\sigma^\mu\\ \bar{\sigma}^\mu&0\end{pmatrix}\begin{pmatrix}-1&0\\0&1\end{pmatrix}\begin{pmatrix}\xi\\ \xi\end{pmatrix}\sqrt{m_N}$$

$$= m_N\, \xi'^\dagger(-\bar{\sigma}^\mu + \sigma^\mu)\,\xi$$

$$= 2m_N\,(0,\ \xi'^\dagger \vec{\sigma}\,\xi)$$

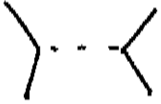
$$q_\mu \bar{u}(p')\gamma^\mu\gamma^5 u(p) = (2m_N)\cdot \; -\vec{q}\cdot(\xi'^\dagger\vec{\sigma}\cdot\xi)$$

so we get the same expression as in the middle of p. 7:

 $= \; i\,(2m_N)^2\, g^2_{\pi NN}\left(\dfrac{-1}{q^2 - m_\pi^2}\right)(\vec{q}\cdot \xi'^\dagger\vec{\sigma}\,\xi_p)(\vec{q}\cdot \xi'^\dagger\vec{\sigma}\cdot\xi_k)$

more generally,

$$\frac{q_\mu}{2m_N}\bar{u}(p')\gamma^\mu\gamma^5 u(p) = -\vec{q}\cdot \xi'^\dagger\vec{\sigma}\cdot\xi = \bar{u}(p')\gamma^5 u(p)$$

so all diagrams of the form  or 

reduce to the same expressions as in the pseudoscalar theory.

f.) Actually, the equation above is exact in the full relativistic theory:

$$\frac{1}{2m_N}\, q_\mu\, \bar{u}(p')\gamma^\mu\gamma^5 u(p)$$

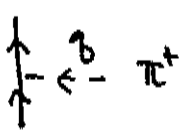
$$= \frac{1}{2m_N}\, \bar{u}(p')\, \not{q}\, \gamma^5\, u(p)$$

$$= \frac{1}{2m_N}\, \bar{u}(p')\, (\not{p}' - \not{p})\gamma^5 u(p)$$

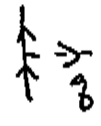
$$= \frac{1}{2m_N}\, \underbrace{\bar{u}(p')\, \not{p}'}_{\bar{u}(p')\cdot m_N} \gamma^5\, u(p) + \bar{u}(p')\gamma^5 \underbrace{\not{p}\, u(p)}_{m_N \cdot u(p)}$$

$$= \bar{u}(p')\gamma^5 u(p)$$

then if all $\underline{p}, \underline{n}$ $\underline{\text{are}}$ $\underline{\text{on}}$-$\underline{\text{shell}}$, the Feynman rules on p. 9 can be rewritten by replacing:



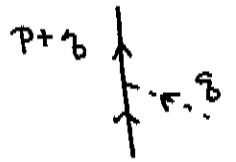
$$= g_{\pi NN}\sqrt{2}\, \frac{\not{q}\gamma^5}{2m_N} = g_{\pi NN}\sqrt{2}\,\gamma^5$$



$$= g_{\pi NN}\sqrt{2}\, \frac{(-\not{q})\gamma^5}{2m_N} = g_{\pi NN}\sqrt{2}\,\gamma^5$$

Then the Feynman rules on p. 9 reduce precisely to those on p. 5.

In diagrams in which there is a fermion line off shell, we find extra terms. If $(p+q)$ is off-shell:



$$= \frac{i}{\not{p}+\not{q}-m_N} \frac{\not{q}}{2m_N} \gamma^5 u(p) \; g_{\pi NN}$$

$$= \frac{i}{\not{p}+\not{q}-m_N} \frac{(\not{p}+\not{q}-\not{p})}{2m_N} \gamma^5 u(p) \cdot g_{\pi NN}$$

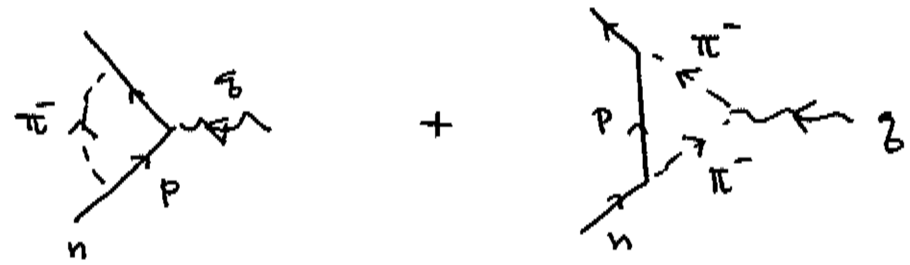
$$= \frac{i}{\not{p}+\not{q}-m_N} \frac{(\not{p}+\not{q}+m_N)}{2m_N} \gamma^5 u(p) \; g_{\pi NN}$$

$$= \frac{i}{\not{p}+\not{q}-m_N} \left(\frac{\not{p}+\not{q}-m_N + 2m_N}{2m_N}\right) \gamma^5 u(p) \; g_{\pi NN}$$

$$= \frac{i}{\not{p}+\not{q}-m_N} \gamma^5 u(p) \; g_{\pi NN} \quad \Big] \text{ result of pseudoscalar theory}$$

$$+ \quad \frac{i}{2m_N} \gamma^5 u(p) \; g_{\pi NN} \quad \Big] \text{ extra term.}$$

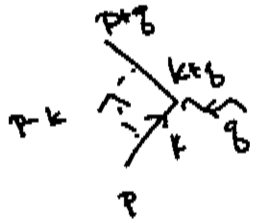
g.) Now consider the electromagnetic form factors of the neutron. These come from:



Since $F_1, F_2 = 0$ in leading order  because the neutron has zero charge, there is no contribution from Z_2 terms.

h.) Now compute the diagrams in the pseudoscalar theory. I will use dimensional regularization assuming $\{\gamma^5, \gamma^\mu\} = 0$ for all μ. (It is not obvious that this is permitted, but it is. See Chapt 19 of Peskin + Schroeder for the explanation.)

Then →



$$= \underset{\longrightarrow}{(+ie)} \int \frac{d^4k}{(2\pi)^4}\, \bar{u}(p')(\sqrt{2}\, g_{\pi NN}\gamma^5)\frac{i(\not{k}+\not{q}+m_N)}{(k+q)^2-m_N^2}\gamma^\mu$$

proton has a + charge.

$$\cdot \frac{i(\not{k}+m_N)}{k^2-m_N^2}\cdot(\sqrt{2}\, g_{\pi NN}\gamma^5)\, u(p)\, \frac{i}{(p-k)^2-m_\pi^2}$$

(anticommute γ^5 to the right)

$$= (+ie)\; 2i g_{\pi NN}^2 \int\frac{d^4k}{(2\pi)^4}\frac{1}{((k+q)^2-m_N^2)(k^2-m_N^2)((p-k)^2-m_\pi^2)}$$

$$\cdot\ \bar{u}(p')(\not{k}+\not{q}-m_N)\gamma^\mu(\not{k}-m_N)\, u(p)$$

$$= (+ie)\; 2i g_{\pi NN}^2 \int dx\,dy\,dz\, \delta(x+y+z-1)\int\frac{d^4\ell}{(2\pi)^4}\frac{1}{[\ell^2-\Delta]^3}$$

$$\cdot\ \bar{u}(p')((\not{k}+\not{q})-m_N)\gamma^\mu(\not{k}-m_N)\, u(p)$$

where the denominator is

$$x((k+q)^2-m_N^2) + y(k^2-m_N^2) + z((k-p)^2-m_\pi^2)$$

$$= k^2 + 2k\cdot(xq-zp) + xq^2 + zp^2 - (1-z)m_N^2 - zm_\pi^2$$

$$= \ell^2 - (xq-zp)^2 + xq^2 + zp^2 - (1-z)m_N^2 - zm_\pi^2$$

$$= \ell^2 + x(1-x)q^2 + z(1-z)p^2 + 2xz\, p\cdot q - (1-z)m_N^2 - zm_\pi^2$$

$$= \ell^2 + xyq^2 + xz(p+q)^2 + yz\,p^2 - (1-z)m_N^2 - zm_\pi^2$$

$$= \ell^2 + xyq^2 - (1-z)^2 m_N^2 - zm_\pi^2$$

so $\Delta = (1-z)^2 m_N^2 + zm_\pi^2 - xyq^2$

$$k = \ell - xq + zp$$
$$k+q = \ell + (1-x)q + zp$$

So

$$= (+ie)\,(2ig^2_{\pi NN}) \int dx\,dy\,dz\,\delta(x+y+z-1) \int \frac{d^4 k}{(2\pi)^4} \frac{2}{[k^2-\Delta]^3}$$

$$\cdot\ \bar{u}(p')\Big[\not{k}\gamma^\mu \not{k} + [\underbrace{(1-x)}_{z+y}\not{q} + z\not{p} - m_N]\gamma^\mu[-x\not{q} + z\not{p} - m_N]\Big] u(p)$$

$$= (+ie)\,(2ig^2_{\pi NN}) \int dx\,dy\,dz\,\delta(x+y+z-1) \int \frac{d^4 k}{(2\pi)^4} \frac{2}{[k^2-\Delta]^3}$$

$$\bar{u}(p')\Big[\not{k}\gamma^\mu \not{k} + \{m_N(1-z) + y\not{q}]\gamma^\mu[-m_N(1-z) - x\not{q}]\Big] u(p)$$

$$= (+ie)\,(2ig^2_{\pi NN}) \int dx\,dy\,dz\,\delta(x+y+z-1)$$

$$\cdot \Big\{ \frac{-i}{(4\pi)^{d/2}} \frac{\Gamma(2-d/2)}{\Delta^{2-d/2}}\ \bar{u}(p')\gamma^\alpha\gamma^\mu\gamma_\alpha u(p) \cdot \frac{1}{2}$$

$$+ \frac{i}{(4\pi)^{d/2}} \frac{\Gamma(3-d/2)}{\Delta^{3-d/2}}\ \bar{u}(p')\{m_N(1-z) + y\not{q}]\gamma^\mu\{m_N(1-z) - x\not{q}] u(p)\Big\}$$

$$\gamma^\alpha\gamma^\mu\gamma_\alpha = 2g^{\mu\alpha}\gamma_\alpha - \gamma^\mu\gamma^\alpha\gamma_\alpha = (2-d)\gamma^\mu$$

$$= (+ie)\,(2ig^2_{\pi NN})\,\frac{i}{(4\pi)^{d/2}}\,\Gamma(3-d/2) \int_0^1 dx\,dy\,dz\,\delta(x+y+z-1)$$

$$\cdot \Big\{ -\frac{2-d}{2}\,\frac{1}{2-d/2}\ \bar{u}(p')\gamma^\mu u(p)\ \frac{1}{\Delta^{2-d/2}}$$

$$+ \bar{u}(p')[m_N(1-z) \blacksquare y\not{q}]\gamma^\mu[m_N(1-z) + x\not{q}] u(p)\ \frac{1}{\Delta^{3-d/2}}\Big\}$$

The second line simplifies as:

$$\bar{u}(p')\,[\;]\,\gamma^\mu\,[\;]\,u(p)$$

$$= \bar{u}(p')\, m_N^2(1-z)^2\,\gamma^\mu\, u(p) - y\, m_N(1-z)\,\bar{u}(p')\,\not{q}\,\gamma^\mu\, u(p)$$

$$+ x\, m_N(1-z)\,\bar{u}(p')\,\gamma^\mu \not{q}\, u(p) - xy\,\bar{u}(p')\,\not{q}\gamma^\mu\not{q}\, u(p)$$

Using $\langle x\rangle = \langle y\rangle$, $\not{q}\gamma^\mu\not{q} = 2q^\mu\not{q} - \gamma^\mu\not{q}\not{q} = (0) - \gamma^\mu q^2$

$$= \bar{u}(p')\,\gamma^\mu\, u(p)\,\left[m_N^2(1-z)^2 + xy\,q^2\right]$$

$$+ x\, m_N(1-z)\,\bar{u}(p')\,[\gamma^\mu, \not{q}]\, u(p)$$

in all:

$$= (+ie)\,\frac{-2g_{\pi NN}^2}{(4\pi)^{d/2}}\,\Gamma(3-d/2)\int dx\,dy\,dz\,\delta(x+y+z-1)$$

$$\left\{ -\frac{2-d}{2}\,\frac{1}{\Delta^{2-d/2}}\,\frac{1}{2-d/2}\;\bar{u}\gamma^\mu u \right.$$

$$\left. + \frac{1}{\Delta^{3-d/2}}\left(\bar{u}\gamma^\mu u\,\left[m_N^2(1-z)^2 + xyq^2\right] + x(1-z)m_N\,\bar{u}\,[\gamma^\mu, \not{q}]\,u\right)\right\}$$

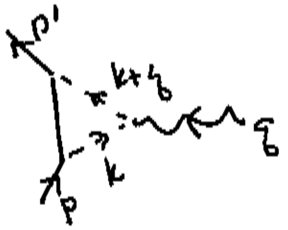
$$= (+ie)\;\bar{u}(p')\left[\gamma^\mu\,\delta_a F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m_N}\,\delta_a F_2(q^2)\right]u(p)$$

$$\delta_a F_1 = \frac{2g_{\pi NN}^2}{(4\pi)^{d/2}}\,\Gamma(3-d/2)\int dx\,dy\,dz\,\delta(x+y+z-1)\left\{\frac{2-d}{2(2-d/2)}\,\frac{1}{\Delta^{2-d/2}}\right.$$

$$\left. - \frac{1}{\Delta^{3-d/2}}\left(m_N^2(1-z)^2 + xy\,q^2\right)\right\}$$

$$\delta_a F_2 = +\frac{2g_{\pi NN}^2}{(4\pi)^{d/2}}\,\Gamma(3-d/2)\int dx\,dy\,dz\,\delta(x+y+z-1)\,\frac{1}{\Delta^{3-d/2}}\,4x(1-z)\,m_N^2$$

Next:



$$= \left(\sqrt{2}\, g_{\pi NN}\right)^2 \int \frac{d^4k}{(2\pi)^4}\; \bar{u}(p')\, \gamma^5\, \frac{i(\not{p}-\not{k}+m_N)}{(p-k)^2-m_N^2}\, \gamma^5\, u(p)$$

$$\cdot \quad \frac{i}{(k+q)^2-m_\pi^2}\quad \underbrace{\left[-ie\,(k+q+k)^\mu\right]}_{\text{Feynman rule from part (a)}}\quad \frac{i}{k^2-m_N^2}$$

$$= (+ie)\,(-2ig_{\pi NN}^2) \int dx\,dy\,dz\,\delta(x+y+z-1) \int \frac{d^4\ell}{(2\pi)^4}\, \frac{2}{[\ell^2-\hat{\Delta}]^3}$$

$$\cdot\ (k+k+q)^\mu \quad \bar{u}(p')\left[(\not{p}-\not{k})-m_N\right] u(p)$$

where the denominator is:

$$x\left((k+q)^2-m_\pi^2\right) + y\left(k^2-m_\pi^2\right) + z\left((k-p)^2-m_N^2\right)$$

$$= \quad \cdots \quad (\text{as on p. } 14)$$

$$= \ell^2 + xyq^2 + \underbrace{xz(p+q)^2 + yz\,p^2}_{z(1-z)\,m_N^2} - (1-z)m_\pi^2 - z m_N^2$$

$$= \ell^2 + xyq^2 - z^2 m_N^2 - (1-z)\,m_\pi^2$$

$$\hat{\Delta} = z^2 m_N^2 + (1-z)\,m_\pi^2 - xyq^2$$

$$k = \ell - xq + zp$$

$$k+q = \ell + (1-x)q + zp$$

$$k-p = \ell - xq - (1-z)p$$

$$= (+ie)(-2ig^2_{\pi NN}) \int dx\,dy\,dz\,\delta(x+y+z-1) \int \frac{d^4k}{(2\pi)^4} \frac{2}{(k^2-\Delta)^3}$$

$$\cdot \Big\{ -2k^\mu k_\alpha \quad \bar{u}(p')\gamma^\alpha u(p)$$

$$+ ((1-2x)q + 2zp)^\mu \quad \bar{u}(p') [x\not{q} + (1-z)\not{p} - m_N] u(p) \Big\}$$

$$\bar{u}\not{q}u = 0 \qquad \bar{u}(p')\not{p}u(p) = m_N \; \bar{u}u$$

$$(1-2x)q + 2zp = 2zp + (z+y-x)q$$

$$= z[p+(p+q)] + \underbrace{(y-x)}_{\langle\,\rangle = 0} q$$

$$= (+ie)(-2ig^2_{\pi NN}) \int dx\,dy\,dz\,\delta(x+y+z-1)$$

$$\Big\{ \frac{-i}{(4\pi)^{d_4}} \frac{\Gamma(2-d_4)}{\Delta^{2-d_4}} (-2)\frac{1}{2} \quad \bar{u}\gamma^\mu u$$

$$+ \frac{i}{(4\pi)^{d_4}} \frac{\Gamma(3-d_4)}{\Delta^{3-d_4}} \cdot z \cdot (p+p')^\mu (-z) m_N \quad \bar{u}u \Big\}$$

by the Gordon id: $\frac{(p+p')^\mu}{2m_N}\bar{u}u = \bar{u}\gamma^\mu u - \bar{u}\frac{i\sigma^{\mu\nu}q_\nu}{2m_N}u$

$$= (+ie)(-2ig_{\pi NN}) \frac{i}{(4\pi)^{d_4}} \Gamma(3-d_4) \int dx\,dy\,dz\,\delta(x+y+z-1)$$

$$\cdot \Big\{ \frac{1}{2-d_4} \bar{u}\gamma^\mu u \frac{1}{\Delta^{2-d_4}}$$

$$- 2z^2 m_N^2 \frac{1}{\Delta^{3-d_4}} \left[\bar{u}\gamma^\mu u - \bar{u}\frac{i\sigma^{\mu\nu}q_\nu}{2m_N} u \right] \Big\}$$

adding the two diagrams:

$$F_1(q^2) = \frac{2 g_{\pi NN}^2}{(4\pi)^{d_L}} \Gamma(3-d_L) \int dx\,dy\,dz\; \delta(x+y+z-1)$$

$$\left\{ \frac{1}{2-d_L} \left(\frac{1}{\Delta^{2-d_L}} \frac{2-d}{2} + \frac{1}{\hat\Delta^{2-d_L}} \right) + \frac{1}{\Delta^{3-d_L}} \left(-m_N^2 (1-z)^2 + xy q^2 \right) + \frac{1}{\hat\Delta^{3-d_L}} (-2z^2 m_N^2) \right\}$$

$$F_2(q^2) = \frac{2 g_{\pi NN}^2}{(4\pi)^{d_L}} \Gamma(3-d_L) \int dx\,dy\,dz\; \delta(x+y+z-1)$$

$$\left\{ - \frac{1}{\Delta^{3-d_L}} (+4x(1-z)\, m_N^2) + \frac{1}{\hat\Delta^{3-d_L}} (2z^2 m_N^2) \right\}$$

$\therefore F_2 \qquad \langle 4x \rangle = \langle 2(x+y) \rangle = 2(1-z)$

$$F_2(q^2) = \frac{2 g_{\pi NN}^2}{(4\pi)^{d_L}} \Gamma(3-d_L) \int_0^1 dz \int_0^{1-z} dx$$

$$\left\{ + \frac{2(1-z)^2 m_N^2}{\Delta^{3-d_L}} + \frac{2z^2 m_N^2}{\hat\Delta^{3-d_L}} \right\}$$

we can take the limit $d \to 4$

$$F_2(q^2) = \frac{2 g_{\pi NN}^2}{(4\pi)^2} \int_0^1 dz \int_0^{1-z} dx$$

$$\left\{ + \frac{2(1-z)^2 m_N^2}{(1-z)^2 m_N^2 + z m_\pi^2 - xy q^2} + \frac{2 z^2 m_N^2}{z^2 m_N^2 + (1-z) m_\pi^2 - xy q^2} \right\}$$

this is UV and IR finite. $F_1(q^2)$ is IR finite,

because Δ, $\hat{\Delta}$ are positive for all x, y, z, $q^2 \leq 0$.

However, there are UV divergent terms:

$$F_1(q^2) = \frac{2 g_{\pi NN}^2}{(4\pi)^2} \int dx\,dy\,dz\, \delta(x+y+z-1) \left\{ \cdot \frac{1}{2-\frac{d}{2}} \left(\frac{2-d}{2} + 1 \right) + \underset{\text{as } d\to 4}{\text{finite}} \right\}$$

$$= \qquad \cdots \qquad \left\{ \frac{1}{2-\frac{d}{2}} \cdot \frac{4-d}{2} + \cdots \right.$$

$$= \qquad \cdots \qquad \left\{ 1 + \text{finite as } d \to 4 \right\}$$

so $F_1(q^2)$ is UV finite.

ii.) $$F_1(q^2=0) = \frac{2 g_{\pi NN}^2}{(4\pi)^{d/2}} \Gamma(3-\tfrac{d}{2}) \int_0^1 dz \int_0^{1-z} dx$$

$$\left\{ \frac{1}{2-d/2} \left(\frac{2-d}{2} \frac{1}{\left[(1-z)^2 m_N^2 + z m_\pi^2\right]^{2-d/2}} + \frac{1}{\left[z^2 m_N^2 + (1-z) m_\pi^2\right]^{2-d/2}} \right) \right.$$

$$\left. - \left(\frac{m_N^2 (1-z)^2}{\left[(1-z)^2 m_N^2 + z m_\pi^2\right]^{3-d/2}} + \frac{2 z^2 m_N^2}{\left[z^2 m_N^2 + (1-z) m_\pi^2\right]^{3-d/2}} \right) \right\}$$

put $\omega = z$ in these terms $\qquad\qquad$ $\omega = (1-z)$ in these terms

$$F_1(q^2=0) = \frac{2g_{\pi NN}^2}{(4\pi)^{d/2}} \Gamma(3-d/2) \int_0^1 d\omega$$

$$\left\{ \frac{1}{2-d/2} \left[\frac{(2-d)}{2}(1-\omega) + \omega \right] \frac{1}{\left[(1-\omega)^2 m_N^2 + \omega m_\pi^2\right]^{2-d/2}} \right.$$

$$\left. - \frac{m_N^2 [(1-\omega)^3 + 2(1-\omega)^2 \omega]}{\left[(1-\omega)^2 m_N^2 + \omega m_\pi^2\right]^{3-d/2}} \right\}$$

$$= \frac{2g_{\pi NN}^2}{(4\pi)^{d/2}} \Gamma(3-d/2) \int_0^1 d\omega$$

$$\left\{ \frac{(1-\omega)}{\left[(1-\omega)^2 m_N^2 + \omega m_\pi^2\right]^{2-d/2}} + \frac{(-(1-2\omega))}{(2-d/2)} \frac{1}{\left[(1-\omega)^2 m_N^2 + \omega m_\pi^2\right]^{2-d/2}} \right.$$

$$\left. - \frac{m_N^2 (1-\omega)^2 (1+\omega)}{\left[[1-\omega]^2 m_N^2 + \omega m_\pi^2\right]^{3-d/2}} \right\}$$

Integrate the middle term by parts:

$$\int_0^1 d\omega \quad -(1-2\omega) \frac{1}{2-d/2} \frac{1}{[\quad]^{2-d/2}}$$

$$= -\frac{\omega(1-\omega)}{2-d/2 \, [\quad]^{2-d/2}} \Bigg|_{\omega=0}^{\omega=1} - \int_0^1 d\omega \frac{-\omega(1-\omega)\left[2(1-\omega) m_N^2 - m_\pi^2\right]}{\left[(1-\omega)^2 m_N^2 + \omega m_\pi^2\right]^{3-d/2}}$$

$$= 0 + \int_0^1 d\omega \frac{2\omega(1-\omega)^2 m_N^2 - \omega(1-\omega) m_\pi^2}{\left[(1-\omega)^2 m_N^2 + \omega m_\pi^2\right]^{3-d/2}}$$

reassemble the pieces:

$$F_1(q^2=0) = \frac{2g_{\pi NN}^2}{(4\pi)^{d_2}}\, \Gamma(3-d_2) \int_0^1 d\omega \, \frac{1}{\left[(1-\omega)^2 m_N^2 + \omega m_\pi^2\right]^{3-d_2}}$$

$$\Big\{ (1-\omega)^3 m_N^2 + (1-\omega)\omega m_\pi^2 + 2\omega(1-\omega)^2 m_N^2 - \omega(1-\omega) m_\pi^2$$

$$- m_N^2 (1-\omega)^2 (1+\omega) \Big\}$$

$$= 0 \;\; !$$ (before taking $d \to 4$)

j.) Now expand $F_1(q^2)$ to order q^2. The coefficient should naturally be finite.

$$F_1(q^2) = \frac{2g_{\pi NN}^2}{(4\pi)^{d_2}}\, \Gamma(3-d_2) \int dx\, dy\, dz\, \delta(x+y+z-1)$$

$$\Big\{ \frac{1}{2-d_2} \frac{1}{\left[(1-z)^2 m_N^2 + z m_\pi^2 - xyq^2\right]^{2-d_2}} \qquad \frac{2-d}{2}$$

$$+ \frac{1}{2-d_2} \frac{1}{\left[z^2 m_N^2 + (1-z) m_\pi^2 - xyq^2\right]^{2-d_2}}$$

$$+ \frac{1}{\left[(1-z)^2 m_N^2 + z m_\pi^2 - xyq^2\right]^{3-d_2}} \left(- m_N^2 (1-z)^2 + xyq^2\right)$$

$$+ \frac{1}{\left[z^2 m_N^2 + (1-z) m_\pi^2 - xyq^2\right]^{3-d_2}} \, 2z^2 m_N^2 \Big\}$$

In the first two terms, we can write

$$\frac{1}{2-\frac{d}{2}}\,\frac{1}{[D-xyq^2]^{2-\frac{d}{2}}} = (\text{const}) - \frac{2-\frac{d}{2}}{2-\frac{d}{2}}\,\frac{-xyq^2}{[D]^{3-\frac{d}{2}}}$$

$$\xrightarrow[d\to 4]{} (\text{const}) + \frac{xyq^2}{D}$$

The last two terms are finite as $d\to 4$; we just expand.

Then the $O(q^2)$ term is

$$F_1(q^2) = \frac{2g_{\pi NN}^2}{(4\pi)^2}\cdot 1\cdot \int dx\,dy\,dz\;\delta(x+y+z-1)$$

$$\Big\{ -1\cdot\frac{xyq^2}{(1-z)^2 m_N^2 + z m_\pi^2} + \frac{xyq^2}{z^2 m_N^2 + (1-z) m_\pi^2}$$

$$+ \frac{xyq^2}{(1-z)^2 m_N^2 + z m_\pi^2} + \frac{(-m_N^2(1-z)^2)\,xyq^2}{[(1-z)^2 m_N^2 + z m_\pi^2]^2}$$

$$+ \frac{(-2z^2 m_N^2)\,xyq^2}{[z^2 m_N^2 + (1-z) m_\pi^2]^2}\Big\}$$

$$= \frac{2g_{\pi NN}^2}{(4\pi)^2}\int dx\,dy\,dz\;\delta(x+y+z-1)\;(xyq^2)$$

$$\cdot\Big[\frac{1}{(1-z)^2 m_N^2 + z m_\pi^2}\Big\{-1+1-1+\frac{z m_\pi^2}{(1-z)^2 m_N^2 + z m_\pi^2}\Big\}$$

$$+ \frac{1}{z^2 m_N^2 + (1-z) m_\pi^2}\Big\{1-2+\frac{2(1-z) m_\pi^2}{z^2 m_N^2 + (1-z) m_\pi^2}\Big\}\Big]$$

$$F_1(q^2) = \frac{2g_{\pi NN}^2}{(4\pi)^2} \; q^2 \cdot \int dx\,dy\,dz\; \delta(x+y+z-1) \cdot \; xy$$

$$\cdot(-1) \left\{ \frac{1}{(1-z)^2 m_N^2 + z m_\pi^2} \;-\; \frac{z m_\pi^2}{[\quad]^2} \right.$$

$$\left. + \; \frac{1}{z^2 m_N^2 + (1-z) m_\pi^2} \;-\; \frac{2(1-z) m_\pi^2}{[\quad]^2} \right\}$$

Now we just have to do the ~~integrals~~.

$$\int_0^{1-z} dx\,dy\; \delta(x+y+z-1)\; xy$$

$$= \int_0^{1-z} dx \;\; x(1-z-x) \;=\; \frac{(1-z)^2}{2}(1-z) - \frac{(1-z)^3}{3} = \frac{(1-z)^3}{6}$$

Now let $\omega = (1-z)$ in the first line above

$\omega = z$ in the second line.

$$= \; - \frac{2g_{\pi NN}^2}{(4\pi)^2} \; \frac{q^2}{6} \int_0^1 d\omega$$

$$\left\{ \frac{\omega^3}{\omega^2 m_N^2 + (1-\omega) m_\pi^2} \;+\; m_\pi^2 \frac{\partial}{\partial m_\pi^2} (\text{this}) \right.$$

$$\left. + \; \frac{(1-\omega)^3}{\omega^2 m_N^2 + (1-\omega) m_\pi^2} \;+\; 2 m_\pi^2 \frac{\partial}{\partial m_\pi^2} (\text{this}) \right\}$$

These are ugly integrals. I'll evaluate them in the limit $m_N \gg m_\pi$. That is, I will drop terms of order $m_\pi/m_N \sim 0.14$.

$$\int_0^1 d\omega \frac{\omega^3}{\omega^2 m_N^2 + (1-\omega) m_\pi^2} \cong \int_0^1 d\omega \; \omega^3 \left\{ \frac{1}{\omega^2 m_N^2} - \frac{m_\pi^2 (1-\omega)}{\omega^4 m_N^4} + \cdots \right\}$$

$$= \frac{1}{2 m_N^2} - \mathcal{O}\left(\frac{m_\pi^2}{m_N^4} \lg m_N^2/m_\pi^2 \right)$$

acts $m_\pi^2 \frac{\partial}{\partial m_\pi^2}$ on this gives $\quad 0 - \mathcal{O}\left(\frac{m_\pi^2}{m_N^4}\right)$

$$\int_0^1 d\omega \frac{(1-\omega)^3}{\omega^2 m_N^2 + (1-\omega) m_\pi^2} = \int_0^1 d\omega \frac{1}{\omega^2 m_N^2 + (1-\omega) m_\pi^2} (1 - 3\omega + 3\omega^2 - \omega^3)$$

Proceed systematically

$$\int_0^1 d\omega \frac{1}{\omega^2 m_N^2 + (1-\omega) m_\pi^2} = \int_0^1 d\omega \frac{1}{\omega^2 m_N^2 + m_\pi^2} + \frac{\omega m_\pi^2}{(\omega^2 m_N^2 + m_\pi^2)^2} + \cdots$$

$$= \frac{1}{m_\pi m_N} \tan^{-1}\left(\frac{m_N}{m_\pi}\right) + \int_0^1 \frac{1}{2} d\omega^2 \frac{m_\pi^2}{[\omega^2 m_N^2 + m_\pi^2]^2} + \cdots$$

$$= \frac{1}{m_\pi m_N}\left(\frac{\pi}{2} - \frac{m_\pi}{m_N} + \cdots\right) + \frac{1}{2 m_N^2} m_\pi^2 \left(\frac{1}{m_\pi^2} - \underbrace{\frac{1}{m_N^2 + m_\pi^2}}_{\mathcal{O}\left(\frac{m_\pi}{m_N}\right)}\right) + \cdots$$

$$= \frac{\pi/2}{m_\pi m_N} - \frac{1}{m_N^2} + \frac{1}{2 m_N^2} + \mathcal{O}\left(\frac{m_\pi}{m_N}\right)$$

$$= \frac{\pi/2}{m_\pi m_N} - \frac{1}{2 m_N^2}$$

but note that

$$\left(1 + 2 m_\pi^2 \frac{\partial}{\partial m_\pi^2}\right)\left(\frac{\pi/2}{m_\pi m_N}\right) = \left(1 + m_\pi \frac{\partial}{\partial m_\pi}\right) \frac{\pi/2}{m_\pi m_N} = 0$$

$$\int_0^1 d\omega \; \frac{\omega}{\omega^2 m_N^2 + (1-\omega) m_\pi^2} \;\cong\; \tfrac{1}{2}\int_0^1 d\omega^2 \; \frac{1}{\omega^2 m_N^2 + m_\pi^2}$$

$$= \frac{1}{2m_N^2} \lg \frac{m_N^2 + m_\pi^2}{m_\pi^2} \;\cong\; \frac{1}{2m_N^2} \lg \frac{m_N^2}{m_\pi^2}$$

$$\left(1 + 2 m_\pi^2 \frac{\partial}{\partial m_\pi^2}\right) \frac{1}{2m_N^2} \lg \frac{m_N^2}{m_\pi^2} = \frac{1}{2m_N^2} \lg \frac{m_N^2}{m_\pi^2} - \frac{1}{2m_N^2} + \cdots$$

$$\int_0^1 d\omega \; \frac{\omega^2}{\omega^2 m_N^2 + (1-\omega) m_\pi^2} \;\approx\; \int_0^1 d\omega \; \frac{1}{m_N^2} = \frac{1}{m_N^2}$$

$$\int_0^1 d\omega \; \frac{\omega^3}{\omega^2 m_N^2 + (1-\omega) m_\pi^2} \;\approx\; \frac{1}{2m_N^2}$$

$\therefore$ all:

$$\left(1 + 2m_\pi^2 \frac{\partial}{\partial m_\pi^2}\right) \int d\omega \; \frac{(1-\omega)^3}{\omega^2 m_N^2 + (1-\omega) m_\pi^2} =$$

$$- \frac{1}{2m_N^2} - \frac{3}{2m_N^2} \lg \frac{m_N^2}{m_\pi^2} + \frac{3}{2m_N^2} + \frac{3}{m_N^2} - \frac{1}{2m_N^2}$$

$$= - \frac{3}{2m_N^2} \lg \frac{m_N^2}{m_\pi^2} + \frac{2}{m_N^2} - \frac{1}{2m_N^2}$$

and the whole bracket on p. 24 becomes.

$$\int_0^1 d\omega \left\{ \; \right\} = - \frac{3}{2m_N^2} \lg \frac{m_N^2}{m_\pi^2} + \frac{2}{m_N^2}$$

then

$$F_1(q^2) = \frac{1}{6} q^2 \cdot \left(- \frac{2 g_{\pi NN}^2}{(4\pi)^2} \right) \left(- \frac{3}{2} \frac{1}{m_N^2} \right)$$

$$\cdot \left(\lg \frac{m_N^2}{m_\pi^2} - \frac{4}{3} \right) \quad \left(1 + O\left(\frac{m_\pi}{m_N}\right)\right)$$

comparing to

$$F_1(q^2) = \frac{1}{6} q^2 r^2$$

we find

$$r^2 = \frac{3 g_{\pi NN}^2}{(4\pi)^2} \frac{1}{m_N^2} \left(\lg \frac{m_N^2}{m_\pi^2} - \frac{4}{3} \right) \quad \left(1 + O\left(\frac{m_\pi}{m_N}\right)\right)$$

put in $\frac{g_{\pi NN}^2}{4\pi} = 14.$ from measurements of $NN \to NN$

$$r^2 = \frac{8.5}{m_N^2} \qquad \text{or} \qquad r = 0.61 \text{ fm}$$

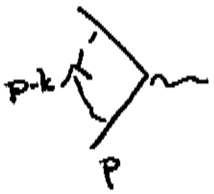
This badly overestimates the experimental result of $r \approx 0.11$ fm

On the other hand, the pion-nucleon model underestimates the neutron magnetic moment

$$g_n = -3.83 \qquad (\text{all "anomalous"})$$

(k) Compare this to the result of the pseudo-vector theory.

I will collect only the divergent contributions to F_1 and F_2:



$$= (+ie) \int \frac{d^4k}{(2\pi)^4}\ \bar{u}(p')\ \sqrt{2} g_{\pi NN} \frac{(\not{p}-\not{k})}{2m_N}\ \gamma^5\ \frac{i(\not{k}+\not{q}+m_N)}{(k+q)^2 - m_N^2}\ \gamma^\mu$$

$$\frac{i(\not{k}+m_N)}{k^2-m_N^2}\ \sqrt{2} g_{\pi NN} \frac{(\not{k}-\not{p})}{2m_N}\ \gamma^5\ u(p)\ \frac{i}{(p-k)^2 - m_\pi^2}$$

$$= (+ie)(2i g_{\pi NN}^2) \int \frac{d^4k}{(2\pi)^4} \frac{1}{[(k+q)^2-m_N^2][k^2-m_N^2][(k-p)^2-m_\pi^2]}$$

$$\bar{u}(p') \frac{(\not{p}-\not{k})}{2m_N} (\not{k}+\not{q}-m_N)\ \gamma^\mu\ (\not{k}-m_N) \frac{(\not{p}-\not{k})}{2m_N}\ u(p)$$

now write

$$\left(\frac{\not{p}-\not{k}}{2m_N}\right) u(p) = \frac{m_N-\not{k}}{2m_N}\ u(p) = \frac{2m_N - (\not{k}+m_N)}{2m_N}\ u(p)$$

$$= \Big(\underbrace{1}_{\text{result in pseudo-scalar theory}} - \underbrace{\frac{\not{k}+m_N}{2m_N}}_{\text{extra term.}} \Big)\ u(p)$$

with the extra term we can write $\displaystyle \frac{\not{k}-m_N}{k^2-m_N^2}\ \not{k}+m_N = 1$,

cancelling a propagator as on p. 12

similarly:

$$\bar{u}(p') \left(\frac{\not{p}-\not{k}}{2m_N}\right) = \bar{u}(p')\ \frac{m_N-\not{k}-\not{q}}{2m_N} = \bar{u}(p') \left[1 - \frac{(\not{k}+\not{q}+m_N)}{2m_N} \right]$$

then

$$= (+ie)\,(2i\,g_{\pi NN}^2)$$

$$\cdot \Big\{ \text{(P-S result)} + \int \frac{d^4k}{(2\pi)^4} \left(-\frac{1}{2m_N}\right) \frac{\bar{u}(p')(\not{k}+\not{q}-m_N)\gamma^\mu u(p)}{[(k+q)^2-m_N^2][(p-k)^2-m_\pi^2]}$$

$$+ \int \frac{d^4k}{(2\pi)^4} \left(-\frac{1}{2m_N}\right) \frac{\bar{u}(p')\gamma^\mu(\not{k}-m_N)\,u(p)}{(k^2-m_N^2)((p-k)^2-m_\pi^2)}$$

$$+ \int \frac{d^4k}{(2\pi)^4} \frac{1}{4m_N^2} \frac{\bar{u}(p')\gamma^\mu u(p)}{(p-k)^2-m_\pi^2} \Big\}$$

These integrals are not so hard, especially if we only need the divergent parts!

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{(p-k)^2-m_\pi^2} = \frac{-i}{(4\pi)^{d/2}} \frac{\Gamma(1-d/2)}{[m_\pi^2]^{1-d/2}}$$

$$\ell = k - zp$$
$$k = \ell + zp$$

$$\int \frac{d^4k}{(2\pi)^4} \frac{\not{k}-m_N}{(k^2-m_N^2)((k-p)^2-m_\pi^2)}$$

$$= \int_0^1 dz \int \frac{d^4\ell}{(2\pi)^4} \frac{\not{\ell}+z\not{p}-m_N}{[\ell^2-\Delta]^2} \qquad \not{p}\,u(p) = m_N u(p)$$

$$= \int_0^1 dz \; \frac{i}{(4\pi)^{d/2}} \frac{\Gamma(2-d/2)}{\Delta^{2-d/2}} \cdot [-(1-z)m_N]$$

$$\Delta = z m_\pi^2 + (1-z)m_N^2 - z(1-z)p^2$$
$$= z m_\pi^2 + (1-z)^2 m_N^2$$

$$\int \frac{d^4k}{(2\pi)^4} \frac{(\not k+\not q - m_N)}{[(k+q)^2-m_N^2][(k-p)^2-m_\pi^2]} \qquad \begin{array}{l} k' = k+q \\ p' = p+q \end{array}$$

$$= \int \frac{d^4k'}{(2\pi)^4}\frac{(\not k' - m_N)}{[k'^2-m_N^2][(k'-p')^2-m_\pi^2]} \quad = \quad \text{previous integral}$$

$$= \int_0^1 dz \, \frac{i}{(4\pi)^{d/2}} \frac{\Gamma(2-d/2)}{\Delta^{2-d/2}} \, (z\not p' - m_N)$$

and, using $\bar u(p')\not p' = \bar u(p') m_N$ $\qquad = \displaystyle\int_0^1 dz \, \frac{i}{(4\pi)^{d/2}} \frac{\Gamma(2-d/2)}{\Delta^{2-d/2}} \left[-(1-z)m_N\right]$

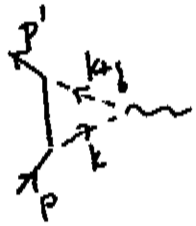
here $\Delta = z m_\pi^2 + (1-z) m_N^2 - z(1-z)p'^2 = z m_\pi^2 + (1-z)^2 m_N^2$ as before.

so all

 $=$ (Pseudoscalar result)

$$+ \quad (+ie)\,(2i g_{\pi NN}^2)\, \frac{i}{(4\pi)^{d/2}}\, \bar u \gamma^\mu u$$

$$\int_0^1 dz \cdot \left\{ -\frac{1}{4m_N^2} \frac{\Gamma(1-d/2)}{[m_\pi^2]^{1-d/2}} - \frac{2}{2m_N} \frac{\Gamma(2-d/2)}{\Delta^{2-d/2}} \left[-(1-z)m_N\right] \right\}$$



$$= (\sqrt{2}\, g_{\pi NN})^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{(k+q)^2 - m_\pi^2} \left[-ie(2k+q)^\mu\right] \frac{i}{k^2 - m_\pi^2}$$

$$\bar{u}(p') \left(\frac{\not{k}+\not{q}}{2m_N}\right) \gamma^5 \frac{i\,(\not{p}-\not{k}+m_N)}{(p-k)^2 - m_N^2} \left(\frac{-\not{k}}{2m_N}\right) \gamma^5 u(p)$$

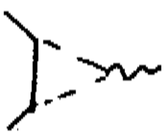
$$= (+ie)(-2i\, g_{\pi NN}^2) \int \frac{d^4k}{(2\pi)^4} \frac{1}{[(k+q)^2 - m_\pi^2][k^2 - m_\pi^2]\ ((k-p)^2 - m_N^2)}$$

$$\cdot\, (2k+q)^\mu \quad \bar{u}(p') \left(\frac{\not{k}+\not{q}}{2m_N}\right) (\not{p}-\not{k}-m_N) \left(\frac{\not{k}}{2m_N}\right) u(p)$$

write $$\frac{\not{k}}{2m_N} u(p) = \left[1 - \frac{\not{p}-\not{k}+m_N}{2m_N}\right] u(p)$$

$$\bar{u}(p') \frac{(\not{k}+\not{q})}{2m_N} = \bar{u}(p) \Big[\underbrace{1}_{\text{P-S result}} - \underbrace{\frac{(\not{p}-\not{k}+m_N)}{2m_N}}_{\text{extra term.}} \Big]$$

so



$$= (+ie)(-2i\, g_{\pi NN}^2) \cdot \Big\{ \text{(Pseudo-scalar result)}$$

$$+ \int \frac{d^4k}{(2\pi)^4} \frac{(2k+q)^\mu}{[(k+q)^2 - m_\pi^2][k^2 - m_\pi^2]} \Big[-\frac{1}{2m_N} - \frac{1}{2m_N}$$

$$+ \frac{1}{4m_N^2} (\not{p}-\not{k}+m_N) \Big] \Big\}$$

Again, the integrals are easy.

$$\int \frac{d^4k}{(2\pi)^4} \frac{(2k+q)^\mu}{[(k+q)^2 - m_\pi^2][k^2 - m_\pi^2]}$$

$\mathbb{k} = k + zq$

$2k+q = \mathbb{k} + (1-2z)q$

$$= \int_0^1 dz \int \frac{d^4\mathbb{k}}{(2\pi)^4} \frac{[2\mathbb{k}^\mu + (1-2z)q^\mu]}{[\mathbb{k}^2 - \Delta]^2}$$

$$= \int_0^1 dz \frac{i}{(4\pi)^{d_2}} \frac{\Gamma(2-d_2)}{[\Delta]^{2-d_2}} [(1-z) - z] q^\mu$$

$\Delta = m_\pi^2 - z(1-z)q^2$ so the integral is antisymmetric under $z \leftrightarrow (1-z)$. so

$$= 0$$

$$\int \frac{d^4k}{(2\pi)^4} \frac{(2k+q)^\mu}{[(k+q)^2 - m_\pi^2][k^2 - m_\pi^2]} (\not{p} - \not{k} + m_N) \cdot$$

$\not{p}\, u(p) = m_N u(p)$

$$= \int_0^1 dz \int \frac{d^4\mathbb{k}}{(2\pi)^4} \frac{[2\mathbb{k}^\mu + (1-2z)q^\mu][2m_N - \not{\mathbb{k}} + z\not{q}]}{[\mathbb{k}^2 - \Delta]^2}$$

$\bar{u}(p')\not{q}\, u(p) = 0$

$$= \int_0^1 dz \frac{-i}{(4\pi)^{d_2}} \cdot (-2) \cdot \frac{\Gamma(1-d_2)}{[\Delta]^{1-d_2}} \frac{1}{2} \gamma^\mu$$

$+$ (terms that are 0 as above)

$$= \int_0^1 dz \frac{i}{(4\pi)^{d_2}} \frac{\Gamma(1-d_2)}{[\Delta]^{1-d_2}} \gamma^\mu$$

so

$$= (ie)(-2i g_{\pi NN}^2) \Big\{ (\text{Pseudo-scalar result})$$

$$+ \int_0^1 dz \; \frac{1}{4m_N^2} \; \frac{i}{(4\pi)^{d/2}} \; \frac{\Gamma(1-d/2)}{[\Delta]^{1-d/2}} \; \bar{u}\gamma^\mu u \Big\}$$

in all

 +  $= (\text{Pseudoscalar result})$

$$+ (ie) \; \frac{2 g_{\pi NN}^2}{(4\pi)^{d/2}} \; \bar{u}(p')\gamma^\mu u(p)$$

$$\int_0^1 dz \; \Big\{ \frac{1}{4m_N^2} \frac{\Gamma(1-d/2)}{[m_\pi^2]^{1-d/2}} + \frac{1}{4m_N^2} \frac{\Gamma(1-d/2)}{[m_\pi^2 - z(1-z)q^2]^{1-d/2}}$$

$$- \frac{\Gamma(2-d/2)}{[z m_\pi^2 + (1-z)^2 m_N^2]^{2-d/2}} \cdot (1-z) \Big\}$$

Note that:

(1) only F_1 is affected, not F_2

(2) this expression is Λ^2 - divergent (pole at $d=2$) at $q^2=0$

(3) the coefficient of q^2 in this expression is $\log \Lambda^2$ divergent.

Now, why doesn't $F_1(0) = 0$ in the pseudovector theory? The answer (which is somewhat beyond the scope of Physics 330) is that the pseudovector theory as I have defined it is not gauge invariant. To make the interaction (7) gauge-invariant, we should have written:

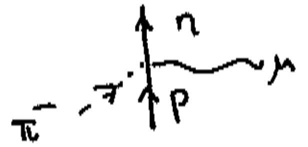
$$\Delta\mathcal{H} = -\frac{g_{\pi NN}}{2m_N} D_\mu \pi^i \; \bar{N}\gamma^\mu\gamma^5\sigma^i N$$

where $D_\mu\pi^- = (\partial_\mu + ieA_\mu)\pi^-$ $\qquad D_\mu\pi^+ = (\partial_\mu - ieA_\mu)\pi^+$

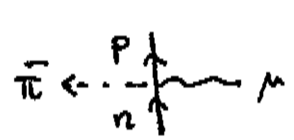
This generates some additional vertices (only in the pseudo-vector theory)

$$\Delta\mathcal{H} = (\text{p.9}) - ieA_\mu \frac{\sqrt{2}\, g_{\pi NN}}{2m_N} \pi^- \; \bar{n}\gamma^\mu\gamma^5 p$$

$$+ ieA_\mu \frac{\sqrt{2}\, g_{\pi NN}}{2m_N} \pi^+ \; \bar{p}\gamma^\mu\gamma^5 n$$



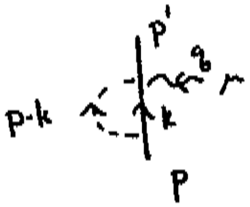
$$= -e\,\frac{\sqrt{2}\, g_{\pi NN}}{2m_N}\gamma^\mu\gamma^5$$



$$= +e\,\frac{\sqrt{2}\, g_{\pi NN}}{2m_N}\gamma^\mu\gamma^5$$

and vertices w. π^+ emission or absorption.

from these, we can built two new diagrams:



$$= \int \frac{d^4k}{(2\pi)^4} \; \bar{u}(p') \left(-e \frac{\sqrt{2} g_{\pi NN}}{2m_N} \gamma^\mu \gamma^5 \right) \frac{i\,\not{k} + m_N}{k^2 - m_N^2}$$

$$\cdot (\sqrt{2} g_{\pi NN}) \left(- \frac{(\not{p} - \not{k})}{2m_N} \right) \gamma^5 \; u(p) \; \cdot \; \frac{i}{(p-k)^2 - m_\pi^2}$$

$$= -e \cdot \frac{2g_{\pi NN}^2}{2m_N} \cdot \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_N^2)((p-k)^2 - m_\pi^2)}$$

$$\cdot \; \bar{u}(p') \, \gamma^\mu \, (\not{k} - m_N) \frac{(\not{p} - \not{k})}{2m_N} \; u(p)$$

$$= (ie) \, (2i g_{\pi NN}^2) \, \frac{1}{2m_N} \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_N^2)((k-p)^2 - m_\pi^2)}$$

$$\cdot \; \bar{u}(p') \, \gamma^\mu (\not{k} - m_N) \left(1 - \frac{\not{k} + m_N}{2m_N} \right) \; u(p)$$

In the second term, a propagator is cancelled. We need the ~~integrals~~:

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{(k-p)^2 - m_\pi^2} \qquad\qquad \int \frac{d^4k}{(2\pi)^4} \frac{\not{k} - m_N}{[k^2 - m_N^2][(k-p)^2 - m_\pi^2]}$$

But, these are already given on p. 29

$$= (ie)\,(2i g^2_{\pi NN})\ \bar{u}\gamma^\mu u \quad \frac{i}{(4\pi)^{d_L}}$$

$$\cdot \int_0^1 dz \left\{ \left[-\frac{(1-z)m_N}{2m_N} \right] \frac{\Gamma(2-d_L)}{[\Delta]^{2-d_L}} + \frac{1}{2m_N}\ \frac{-1}{2m_N}\,(-1)\,\frac{\Gamma(1-d_L)}{[m_\pi]^{1-d_L}} \right\}$$

similarly:

$$= \int \frac{d^4k}{(2\pi)^4}\ \bar{u}(p')\ \sqrt{2}\, g_{\pi NN} \left(\frac{\not{p}-\not{k}}{2m_N} \right) \gamma^5 \frac{i\,(\not{k}+\not{q}-m_N)}{(k+q)^2-m_N^2}$$

$$\cdot \left(+e\, \frac{\sqrt{2}\, g_{\pi NN}}{2m_N} \gamma^\mu \gamma^5 \right) u(p)\ \frac{i}{(p-k)^2-m_\pi^2}$$

$$= (ie)\ 2i g^2_{\pi NN}\ \frac{1}{2m_N} \int \frac{d^4k}{(2\pi)^4}\ \frac{1}{(k+q)^2-m_N^2\ ((k-p)^2-m_\pi^2)}$$

$$\bar{u}(p')\left(1 - \frac{\not{k}+\not{q}+m_N}{2m_N}\right)(\not{k}+\not{q}-m_N)\,\gamma^\mu\, u(p)$$

$$= (ie)\,(2i g^2_{\pi NN})\ \bar{u}\gamma^\mu u \cdot \frac{i}{(4\pi)^{d_L}}$$

$$\cdot \int_0^1 dz \left\{ \left(-\frac{(1-z)m_N}{2m_N} \right) \frac{\Gamma(2-d_L)}{\Delta^{2-d_L}} + \frac{-1}{(2m_N)^2}(-1)\,\frac{\Gamma(1-d_L)}{[m_\pi^2]^{1-d_L}} \right\}$$

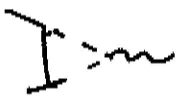
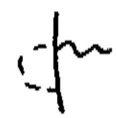
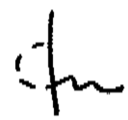
in all

 + 

$$= (ie)\,\frac{2 g^2_{\pi NN}}{(4\pi)^{d/2}}\,\bar{u}(p')\gamma^\mu u(p)$$

$$\cdot \int_0^1 dz \left\{ -\frac{2}{4m_N^2}\,\frac{\Gamma(1-d/2)}{[m_\pi^2]^{1-d/2}} + \frac{\Gamma(2-d/2)}{[\;\Delta\;]^{2-d/2}}\,(1-z) \right\}$$

add this to p.33:

 +  +  + 

$$= \text{(Pseudoscalar result.)}$$

$$+ (ie)\,(\bar{u}\gamma^\mu u)\,\frac{2 g^2_{\pi NN}}{(4\pi)^{d/2}}$$

$$\cdot \int_0^1 dz\; \frac{1}{4m_N^2}\,\Gamma(1-d/2)\left[\frac{1}{[m_\pi^2 - z(1-z)q^2]^{1-d/2}} - \frac{1}{[m_\pi^2]^{1-d/2}} \right]$$

Excellent! This vanishes at $q^2 = 0$, as required.

The expression is regular for $d<4$. As $d \to 4$, there is a pole:

$$\Delta F_1(q^2) = \frac{2g_{\pi NN}^2}{(4\pi)^{d/2}} \frac{1}{(1-d/2)} \frac{1}{(4m_N^2)} \Gamma(2-d/2)$$

$$\int_0^1 dz \cdot \left\{ \frac{m_\pi^2 - z(1-z)q^2}{[m_\pi^2 - z(1-z)q^2]^{2-d/2}} - \frac{m_\pi^2}{[m_\pi^2]^{2-d/2}} \right\}$$

$$\xrightarrow[d\to 4]{} \quad \frac{2g_{\pi NN}^2}{(4\pi)^2} \frac{1}{(-1)} \frac{1}{4m_N^2} \left(\frac{1}{2-d/2}\right) \int_0^1 dz\, (-z(1-z)q^2)$$

$$\Delta F_1(q^2) \sim \frac{1}{6} \cdot \frac{g_{\pi NN}^2}{32\pi^2} \frac{q^2}{m_N^2} \lg \Lambda^2/m_\pi^2$$

so, even after the cancellation, $F_1(q^2)$ is divergent in the pseudo-vector theory.

The calculation of $F_1(q^2)|_{PV} - F_1(q^2)|_{PS}$ involves no difficult integrals. So it is not unlikely that Feynman could do this calculation in one night !